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Higher-order feasible building blocks for lattice structure of oversampled linear-phase perfect reconstruction filter banks ☆

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ABSTRACT

This paper proposes new building blocks for the lattice structure of oversampled linear-phase perfect reconstruction filter banks (OLPPRFBs). The structure is an extended version of higher-order feasible building blocks for critically sampled LPPRFBs. It uses fewer number of building blocks and design parameters than those of traditional OLPPRFBs, whereas frequency characteristics of the new OLPPRFBs are comparable to those of traditional one. Furthermore, the building block structures for arbitrary number of channels and downsampling factors are presented.

1. Introduction

Oversampled filter bank (OFB) [1,2] is one of the most developing topics in the field of FB theory. A typical structure of P-channel OFBs with a downsampling factor of M ($P>M$) and its polyphase representation are illustrated in Fig. 1. OFB is a superclass of critically sampled FBs since its polyphase matrix has the size $P\times M$ which covers the entire $M\times M$ class for critically sampled ones. They have more design freedom than the critically sampled FBs, thus they are expected to have good frequency characteristics and/or robustness against noise [3–5]. Additionally, linear-phase (LP) and perfect reconstruction (PR) properties are usually highly desirable in practical signal processing frameworks [6,7]. In this paper, we present a new lattice structure of oversampled linear-phase perfect reconstruction filter banks (OLPPRFBs).

The lattice structure is based on a factorization of polyphase matrices of FBs. The lattice structure of OLPPRFBs with rational (arbitrary P/M) oversampling ratio was proposed by Labeau et al. for the paraunitary (PU) case [8], and they have also shown the necessary existence conditions. Then, Gan and Ma investigated lattice structures for OLPPRFBs [9,10] in both PU and biorthogonal (BO) cases, and proved the necessary conditions which are regarded as the extended version of those for critically sampled LPPRFBs. The similar lattice structures to Gan's OLPPRFBs were also shown in [11–13].

If OLPPRFBs with long filter length (high-order) are desired for better frequency characteristics such as stopband attenuation, they can be realized by cascading lower-ordered building blocks. Usually order-1 (the highest order of z^{-1} is one) building blocks are used as the lowest order [8–14]. The lattice structure of cascaded order-1 building blocks is effective in the viewpoint of achieving any-order FBs. However, the high-order FB

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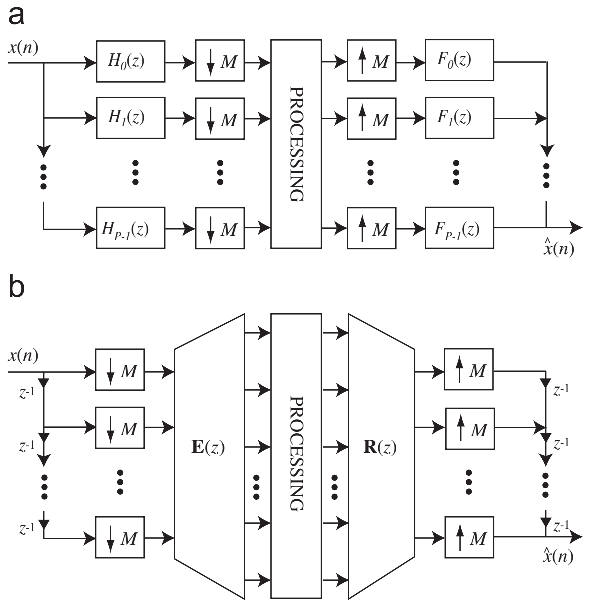

Fig. 1. P-channel OFB with decimation factor M. (a) Conventional representation. (b) Polyphase representation.

requires many order-1 building blocks which increases the implementation cost. Therefore, finding the efficient structure of order-N building blocks is an interesting research problem. Generally it is very complicated since an order-N building block is composed of a matrix polynomial of delay elements z^{-i} (i: some integer). The problem is to guarantee the LP property and calculate its inverse with a reasonable cost for the synthesis bank.

The authors proposed a lattice factorization of critically sampled BO LPFBs [15,16] called higher-order feasible (HOF) building block. It realizes an order-N ($N \geqslant 2$) LPPRFB with one HOF building block. Additionally, it is also regarded as an extended class of traditional (critically sampled) order-1 building blocks [17–19]. Under the same filter length, LPPRFBs based on HOF building blocks realize comparable performance to those based on cascaded order-1 ones, in spite of having fewer number of building blocks and design parameters.

We extend the HOF building block from critically sampled to oversampled LPPRFBs. It inherits all the merits from the critically sampled HOF structure. Furthermore, it provides a new structure of a $P \times M$ rectangular oversampling matrix different from the other building blocks. Our new structure has feasibility for any P and M, thus it is an extension of traditional order-1 building blocks of OLPPRFBs. Moreover, parameterizations for both PR and PU cases are described in the paper, which are generalized methods for the HOF building blocks from the critically sampled case to the $P \geqslant M$ case. This paper provides all possible solutions for $P \geqslant M$ and both BO and PU paramaterizations, whereas the previously proposed HOF structure [15,16] is focused on the conditions under BO and $P = M$. Thus, the proposed structure can be regarded as not only a generalization of the traditional OLPPRFBs but also a broadened class of critically sampled HOF structure. The oversampled HOF building block could change the size of a polyphase matrix from $M \times M$ to $P \times M$ *as well as* having the higher-order structure. The previous oversampled structure cannot have such feature. In the design examples, our OLPPRFBs based on HOF building blocks show comparable frequency characteristics to those based on the cascaded order-1 structure in spite of having fewer number of design parameters.

The rest of this paper is organized as follows: Section 2 reviews traditional lattice structures of OLPPRFBs and critically sampled HOF building blocks. Section 3 describes Type 1 oversampled HOF structure which corresponds to the case that the number of symmetric and antisymmetric filters is the same. The structure of HOF building blocks for Type 2 OLPPRFBs, which belong to the case that the number of symmetric filters is different from that of antisymmetric ones, is presented in Section 4. Section 5 validates our proposed structure by comparing the performance with the traditional ones. Finally, the paper is concluded in Section 6.

Notations: $\lceil\cdot\rceil$ and $\lfloor\cdot\rfloor$ define the functions which return ceiling and floor of $(\cdot)$, respectively. We also define the special matrices of size $S \times S$; $\mathbf{I}_S$ is the identity matrix, $\mathbf{J}_S$ is the reversal identity matrix and $\mathbf{0}_S$ is the null matrix. If they are obvious, the sizes may be omitted for simple representation. Finally, a $P \times M$ rectangular matrix $\mathbf{A}$ will be called *invertible* if an $M \times P$ matrix $\mathbf{X}$ satisfies $\mathbf{XA} = \mathbf{I}_M$ is existed. In the special case $\mathbf{X} = \mathbf{A}^T$, $\mathbf{A}$ will be called *orthogonal*.

2. Review

2.1. General lattice structure of OLPPRFBs

Consider a P-channel OLPPRFB with its filter length $L = KM$ (denoted $P \times KM$, hereafter), N_s symmetric filters, N_a antisymmetric ones, and a downsampling factor of M ($P > M$). The lattice structure is represented as follows [9,10]:

$$\mathbf{E}(z) = \mathbf{G}_{K-1}(z)\mathbf{G}_{K-2}(z)\ldots\mathbf{G}_1(z)\mathbf{E}_0(z) \tag{1}$$

where $\mathbf{G}_i(z)$ ($i = 1, 2, \ldots, K-1$) and $\mathbf{E}_0(z)$ have sizes of $P \times P$ and $P \times M$, respectively. If PR is achieved, the causal synthesis polyphase matrix $\mathbf{R}(z)$ is given by

$$\mathbf{R}(z) = z^{-(K-1)}\mathbf{E}_0^{-1}(z)\mathbf{G}_1^{-1}(z)\mathbf{G}_2^{-1}(z)\ldots\mathbf{G}_{K-1}^{-1}(z) \tag{2}$$

where $\mathbf{E}_0^{-1}(z)$ is an $M \times P$ left-inverse matrix of $\mathbf{E}_0(z)$. We also introduce the types of OLPPRFBs described in [10]:

- Type 1 OLPPRFBs: $N_s = N_a$, the order of $\mathbf{G}_i(z) = 1$.
- Type 2 OLPPRFBs: $N_s \neq N_a$, the order of $\mathbf{G}_i(z) \neq 1$.

Note that all odd-channel OLPPRFBs are categorized in Type 2, whereas the even-channel ones could be both types. OLPPRFBs could have wider ranges about N_s and N_a than the critically sampled ($P = M$) case which is summarized in Table 1. For the simplicity purpose, we assume $N_s \geqslant N_a$ hereafter. However, the $N_s < N_a$ case can be easily extended. The lattice components in P-channel

Table 1
Necessary and sufficient constraints for P-channel OLPPRFBs with sampling factor M $(P>M)$ and filter length $L=KM$.

M	K	Symmetry polarity
Even	Arbitrary	$\frac{M}{2}\leqslant N_s\leqslant P-\frac{M}{2},\ \frac{M}{2}\leqslant N_a\leqslant P-\frac{M}{2}$
Odd	Odd	$\frac{M+1}{2}\leqslant N_s\leqslant P-\frac{M-1}{2},\ \frac{M-1}{2}\leqslant N_a\leqslant P-\frac{M+1}{2}$
	Even	$\frac{M+1}{2}\leqslant N_s\leqslant P-\frac{M+1}{2},\ \frac{M+1}{2}\leqslant N_a\leqslant P-\frac{M+1}{2}$

Type 1 OLPPRFBs are formulated as follows:

$$\mathbf{G}_i(z)=\begin{bmatrix}\mathbf{U}_i & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{P/2}\end{bmatrix}\mathbf{W}_P\boldsymbol{\Lambda}(z)\mathbf{W}_P \tag{3}$$

$$\mathbf{E}_0(z)=\begin{bmatrix}\mathbf{U}_0 & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_0\end{bmatrix}\mathbf{W}_M\begin{bmatrix}\mathbf{I}_{\lceil M/2\rceil} & \mathbf{0}\\ \mathbf{0} & \mathbf{J}_{\lfloor M/2\rfloor}\end{bmatrix} \tag{4}$$

where

$$\mathbf{W}_S=\begin{cases}\frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{I}_s & \mathbf{I}_s\\ \mathbf{I}_s & -\mathbf{I}_s\end{bmatrix}, & S=2s\\ \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{I}_s & \mathbf{0} & \mathbf{I}_s\\ \mathbf{0} & \sqrt{2} & \mathbf{0}\\ \mathbf{I}_s & \mathbf{0} & -\mathbf{I}_s\end{bmatrix}, & S=2s+1\end{cases}$$

$$\boldsymbol{\Lambda}(z)=\begin{bmatrix}z^{-1}\mathbf{I}_{P/2} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{P/2}\end{bmatrix}$$

The size of $\mathbf{U}_i$ is $P/2\times P/2$, and those of $\mathbf{U}_0$ and $\mathbf{V}_0$ are $P/2\times\lceil M/2\rceil$ and $P/2\times\lfloor M/2\rfloor$, respectively. If all of these matrices are orthogonal, the resulting FB is an OLPPUFB. In the PU system, $\mathbf{U}_i$'s are parameterized by using the popular plane rotation method [6,20]. Furthermore, rectangular orthogonal matrices $\mathbf{U}_0$ and $\mathbf{V}_0$ are completely constructed with the extension of the plane rotation for rectangular matrices [21]. If all the matrices are invertible, the resulting FB is a general OLPPRFBs. For the PR case, the rectangular matrices $\mathbf{U}_0$ and $\mathbf{V}_0$ are often parameterized with the singular value decomposition (SVD) [11,12] or the LDU decomposition [10]. Generally, their left-inverse matrices are obtained by using Moore–Penrose pseudo-inverse method. In contrast, the LDU decomposition provides nonunique left-inverse matrices which lead to the possibility of flexible design for the synthesis bank.

$\mathbf{G}_i(z)$ of Type 2 OLPPRFBs is represented as follows:

$$\mathbf{G}_i(z)=\begin{bmatrix}\mathbf{U}_{i,1} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{N_a}\end{bmatrix}\widetilde{\mathbf{W}}\boldsymbol{\Lambda}_1(z)\widetilde{\mathbf{W}}\begin{bmatrix}\mathbf{U}_{i,0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{N_s}\end{bmatrix}\widetilde{\mathbf{W}}\boldsymbol{\Lambda}_0(z)\widetilde{\mathbf{W}} \tag{5}$$

where $\mathbf{U}_{i,1}$ and $\mathbf{U}_{i,0}$ are $N_s\times N_s$ and $N_a\times N_a$ nonsingular matrices, respectively, and $\widetilde{\mathbf{W}}$ is a constant matrix expressed as

$$\widetilde{\mathbf{W}}=\frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{I}_{N_a} & \mathbf{0} & \mathbf{I}_{N_a}\\ \mathbf{0} & \sqrt{2}\mathbf{I}_{N_s-N_a} & \mathbf{0}\\ \mathbf{I}_{N_a} & \mathbf{0} & -\mathbf{I}_{N_a}\end{bmatrix} \tag{6}$$

The delay terms take the forms of $\boldsymbol{\Lambda}_1(z)=\mathrm{diag}(z^{-1}\mathbf{I}_{N_s},\mathbf{I}_{N_a})$ and $\boldsymbol{\Lambda}_0(z)=\mathrm{diag}(z^{-1}\mathbf{I}_{N_a},\mathbf{I}_{N_s})$. Furthermore, $\mathbf{E}_0(z)$ for Type 2 OLPPRFBs depends on K. If K is odd, $\mathbf{E}_0(z)$ is the same as (4) but the sizes of $\mathbf{U}_0$ and $\mathbf{V}_0$ are $N_s\times\lceil M/2\rceil$ and $N_a\times\lfloor M/2\rfloor$, respectively. If K is even, the order of $\mathbf{E}_0(z)$ has to be 1, thus it is represented as follows:

$$\mathbf{E}_0(z)=\begin{bmatrix}\mathbf{U}_{0,1} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{N_a}\end{bmatrix}\mathbf{W}_{2N_a}\boldsymbol{\Lambda}(z)\mathbf{W}_{2N_a}\begin{bmatrix}\mathbf{U}_{0,0} & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_0\end{bmatrix}\times\mathbf{W}_M\begin{bmatrix}\mathbf{I}_{\lceil M/2\rceil} & \mathbf{0}\\ \mathbf{0} & \mathbf{J}_{\lfloor M/2\rfloor}\end{bmatrix} \tag{7}$$

where $\mathbf{U}_{0,1}$, $\mathbf{U}_{0,0}$ and $\mathbf{V}_0$ are $N_s\times N_a$, $N_a\times\lceil M/2\rceil$ and $N_a\times\lfloor M/2\rfloor$ nonsingular matrices, respectively.

2.2. Pre- and postprocessing based lattice structure for Type 1 OLPPRFBs

In [14], a subset in the general lattice structure of Type 1 OLPPRFBs was proposed. It can be realized to implement a critically sampled M-channel LPPRFB by applying to a general P-channel OLPPRFB as a preprocessing system. Its lattice structure for even M takes the form

$$\mathbf{E}(z)=\mathbf{G}_{K_1}(z)\mathbf{G}_{K_1-1}(z)\ldots\mathbf{G}_1(z)\bar{\mathbf{E}}_0(z)\mathbf{Q}_{K_0}(z)\mathbf{Q}_{K_0-1}(z)\ldots\mathbf{Q}_1(z)\mathbf{Q}_0 \tag{8}$$

where $K_0+K_1=K-1$ and

$$\bar{\mathbf{E}}_0(z)=\begin{bmatrix}\bar{\mathbf{U}}_0 & \mathbf{0}\\ \mathbf{0} & \bar{\mathbf{V}}_0\end{bmatrix}\mathbf{W}_M\boldsymbol{\Lambda}(z)\mathbf{W}_M \tag{9}$$

$$\mathbf{Q}_j(z)=\begin{bmatrix}\mathbf{U}_j & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{M/2}\end{bmatrix}\mathbf{W}_M\boldsymbol{\Lambda}(z)\mathbf{W}_M \tag{10}$$

$$\mathbf{Q}_0=\begin{bmatrix}\mathbf{U}_0 & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{M/2}\end{bmatrix}\mathbf{W}_M\begin{bmatrix}\mathbf{I}_{M/2} & \mathbf{0}\\ \mathbf{0} & \mathbf{J}_{M/2}\end{bmatrix} \tag{11}$$

In the above equations, $\bar{\mathbf{U}}_0$, $\bar{\mathbf{V}}_0$ and $\mathbf{U}_j$ are $P/2\times M/2$, $P/2\times M/2$ and $M/2\times M/2$ nonsingular matrices, respectively. The structure can be represented as a pre- and post-processing system of an oversampling building block $\bar{\mathbf{E}}_0(z)$. Obviously, it does not cover all classes of the general Type 1 OLPPRFBs shown in the previous subsection. However, it requires fewer number of delay elements and design parameters than Gan's structure. This is a trade-off issue between implementation cost and filter performance. We consider a new lattice structure of OLPPRFBs based on the pre- and postprocessing system.

2.3. Lattice structure of critically sampled BOLPFBs with HOF building blocks

In [15,16], a lattice structure of critically sampled BOLPFBs which realize various filter length patterns with one building block was proposed. The order-N building block for even-channel BOLPFBs is represented as follows:

$$\mathbf{G}^e_{HOF}(z)=\begin{bmatrix}\mathbf{U} & \mathbf{0}\\ \mathbf{0} & \mathbf{V}\end{bmatrix}\mathbf{W}_M\boldsymbol{\Lambda}^e_{HOF}(z)\mathbf{W}_M \tag{12}$$

where the $\mathbf{U}$ and $\mathbf{V}$ are $M/2 \times M/2$ nonsingular matrices, respectively, and

$$\mathbf{\Lambda}^{e}_{HOF}(z) = \begin{bmatrix} \hat{\mathbf{\Lambda}}_{HOF}(z) & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{\Lambda}}'_{HOF}(z) \end{bmatrix} \tag{13}$$

Besides, $\hat{\mathbf{\Lambda}}_{HOF}(z)$ is an $M/2 \times M/2$ nonsingular matrix whose every element is a *monomial* of z. $\hat{\mathbf{\Lambda}}'_{HOF}(z)$ is the same as $\hat{\mathbf{\Lambda}}_{HOF}(z)$ except powers of z. In (13), the constant matrices $\hat{\mathbf{\Lambda}}_{HOF}(1)$ and $\hat{\mathbf{\Lambda}}'_{HOF}(1)$ take the form

$$\hat{\mathbf{\Lambda}}_{HOF}(1) = \hat{\mathbf{\Lambda}}'_{HOF}(1) = \mathbf{LR} \tag{14}$$

where

$$\mathbf{L} = \begin{bmatrix} \mathbf{L}_0 & \mathbf{0} & \\ \mathbf{0} & \mathbf{L}_1 & \ddots \\ & \ddots & \ddots \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} \mathbf{R}_0 & \mathbf{0} & \\ \mathbf{0} & \mathbf{R}_1 & \ddots \\ & \ddots & \ddots \end{bmatrix}. \tag{15}$$

$\mathbf{L}_n$ and $\mathbf{R}_n$ $(n = 0, 1, \ldots)$ are $\tilde{N} \times \tilde{N}$ $(\tilde{N} \leqslant (N+1)/2)$ lower and upper triangular matrices whose diagonal elements are all 1, respectively. Furthermore, $s_{i,j}$ and $s'_{i,j}$ are the power of z in the (i,j)-th element of $\hat{\mathbf{\Lambda}}_{HOF}(z)$ and $\hat{\mathbf{\Lambda}}'_{HOF}(z)$, respectively:

$$s_{i,j} = \begin{cases} s_{i,i} = \dfrac{-(N+1)}{2}, \\ s_{i,j+1} = s_{i,j} + 1, \\ s_{i+1,j} = s_{i,j} - 1, \end{cases} \quad s'_{i,j} = s_{j,i} + 1 \tag{16}$$

They are determined based on the requirements to yield good filter characteristics as follows:

(1) All diagonal elements in $\hat{\mathbf{\Lambda}}_{HOF}(z)$ should have the same powers.
(2) The difference of the powers of z between the diagonal elements in $\hat{\mathbf{\Lambda}}_{HOF}(z)$ and those in $\hat{\mathbf{\Lambda}}'_{HOF}(z)$ should be $|1|$.
(3) Powers of z in $\hat{\mathbf{\Lambda}}_{HOF}(z)$ and those in $\hat{\mathbf{\Lambda}}'_{HOF}(z)$ should be changed gradually.

The structure of $\mathbf{G}^{e}_{HOF}(z)$ is shown in Fig. 2 (shown for $M = 6$, order-5). The inverse of $\mathbf{G}^{e}_{HOF}(z)$ for the synthesis bank is also represented as

$$\mathbf{G}^{e^{-1}}_{HOF}(z) = \mathbf{W}_M \mathbf{\Lambda}^{e}_{IHOF}(z) \mathbf{W}_M \begin{bmatrix} \mathbf{U}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{V}^{-1} \end{bmatrix} \tag{17}$$

$\mathbf{\Lambda}^{e}_{IHOF}(z)$ is formulated as

$$\mathbf{\Lambda}^{e}_{IHOF}(z) = \begin{bmatrix} \hat{\mathbf{\Lambda}}_{IHOF}(z) & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{\Lambda}}'_{IHOF}(z) \end{bmatrix} \tag{18}$$

where $\hat{\mathbf{\Lambda}}_{IHOF}(1) = \hat{\mathbf{\Lambda}}'_{IHOF}(1) = \mathbf{R}^{-1}\mathbf{L}^{-1}$ and the powers of z in $\hat{\mathbf{\Lambda}}_{IHOF}(z)$ and $\hat{\mathbf{\Lambda}}'_{IHOF}(z)$ are determined by $s'_{j,i}$ and $s_{j,i}$, respectively.

Furthermore, the odd-channel HOF building block is represented as

$$\mathbf{G}^{o}_{HOF}(z) = \begin{bmatrix} \mathbf{U}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{\lfloor M/2 \rfloor} \end{bmatrix} \mathbf{W}_M \begin{bmatrix} z^{-1}\mathbf{I}_{\lfloor M/2 \rfloor} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{\lceil M/2 \rceil} \end{bmatrix} \mathbf{W}_M \times \begin{bmatrix} \mathbf{Q}_{i,0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{i,1} \end{bmatrix} \mathbf{W}_M \mathbf{\Lambda}^{o}_{HOF}(z) \mathbf{W}_M \tag{19}$$

where the $\mathbf{U}_i$, $\mathbf{Q}_{i,0}$ and $\mathbf{Q}_{i,1}$ are $\lceil M/2 \rceil \times \lceil M/2 \rceil$, $\lfloor M/2 \rfloor \times \lfloor M/2 \rfloor$ and $\lceil M/2 \rceil \times \lceil M/2 \rceil$ nonsingular matrices, respectively. $\mathbf{\Lambda}^{o}_{HOF}(z)$ takes the form

$$\mathbf{\Lambda}^{o}_{HOF}(z) = \begin{bmatrix} \hat{\mathbf{\Lambda}}_{HOF}(z) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & z^{-(N-1)/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \hat{\mathbf{\Lambda}}'_{HOF}(z) \end{bmatrix} \tag{20}$$

where both $\hat{\mathbf{\Lambda}}_{HOF}(z)$ and $\hat{\mathbf{\Lambda}}'_{HOF}(z)$ have the size $(M-1)/2 \times (M-1)/2$. For the HOF building block, a starting lattice block $\mathbf{E}_0$ has the same structure as (4) except the sizes of $\mathbf{U}_0$ and $\mathbf{V}_0$ which are $\lceil M/2 \rceil \times \lceil M/2 \rceil$ and $\lfloor M/2 \rfloor \times \lfloor M/2 \rfloor$, respectively.

For both even and odd-channel cases, if $\mathbf{L} = \mathbf{R} = \mathbf{I}_{\lfloor M/2 \rfloor}$, the resulting building block is the same as that of the traditional LPPRFBs [18]. Hence the HOF structure covers the traditional order-1 structure.

3. HOF building blocks for Type 1 OLPPRFBs

In this section, we describe the extension of the HOF building blocks from the critically sampled case to the oversampled one for Type 1 OLPPRFBs. The parameterizations of oversampled HOF building blocks are also shown. First of all, we assume new OLPPRFBs are factorized as follows:

$$\mathbf{E}(z) = \mathbf{G}_{K_1}(z) \ldots \mathbf{G}_1(z) \widetilde{\mathbf{E}}_0(z) \mathbf{Q}_{K_0}(z) \ldots \mathbf{Q}_1(z) \widetilde{\mathbf{Q}}_0, \tag{21}$$

where $\widetilde{\mathbf{E}}_0(z)$ is an oversampling building block and $\widetilde{\mathbf{Q}}_0$ is supposed to be the similar structure to (11). In this

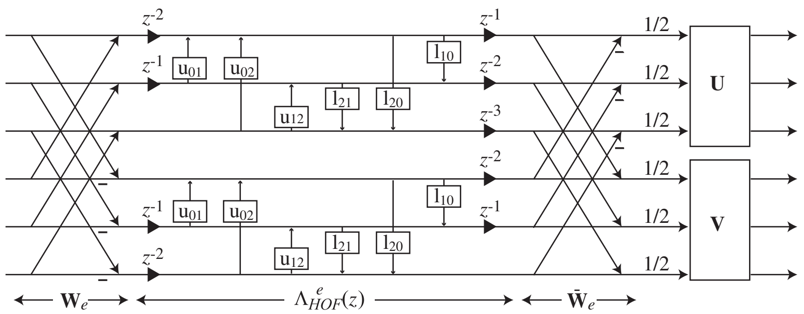

Fig. 2. Structure of even-channel analysis HOF building block (shown for $M = 6$ and order-5), where u_{ij} and l_{ij} are the coefficients in $\mathbf{L}_0$ and $\mathbf{R}_0$, respectively.

structure, an oversampled HOF building block is applied at $\widetilde{\mathbf{E}}_0(z)$, and we consider OLPPRFBs with the same filter length for both analysis and synthesis banks.

3.1. Structure of HOF building blocks for Type 1 OLPPRFBs

In order to factorize $\widetilde{\mathbf{E}}_0(z)$, we assume that it is factorized into the similar structure to (9) shown as

$$\widetilde{\mathbf{E}}_0(z) = \begin{bmatrix} \widetilde{\mathbf{U}}_0 & \mathbf{0} \\ \mathbf{0} & \widetilde{\mathbf{V}}_0 \end{bmatrix} \mathbf{W} \boldsymbol{\Lambda}_{OH}(z) \mathbf{W}$$

There could be two patterns for the oversampling rectangular matrix such as

- Pattern 1: oversampling at the block diagonal matrix $\mathrm{diag}(\widetilde{\mathbf{U}}_0, \widetilde{\mathbf{V}}_0)$.
- Pattern 2: oversampling at the delay matrix $\boldsymbol{\Lambda}_{OH}(z)$.

It is obvious that Pattern 2 is totally different from the other lattice structures since the traditional ones cannot be permitted to have the Pattern 2 and thus, we introduce such structure of $\boldsymbol{\Lambda}_{OH}(z)$. Although Pattern 1 is not a core system of oversampled HOF building blocks, it helps to construct the solutions for any P and M in Type 2 OLPPRFBs, which is described latter in the next section.

First, we consider the simplest approach where the size of $\boldsymbol{\Lambda}_{OH}(z)$ is assumed to be $P \times M$. However, note that M would be even or odd and Type 1 OLPPRFBs could have any K [9,10]. Therefore, the propagating lattice blocks $\mathbf{G}_i(z)$ and $\mathbf{Q}_j(z)$ have to have order-1. These are the critically sampled ones, hence only those for the even-channel case, (3) and (10), satisfy the requirement. In order to adjust the size of the polyphase matrix for that case, let the initial lattice block $\widetilde{\mathbf{Q}}_0$ be a matrix of the size $2\lceil M/2 \rceil \times M$ represented as

$$\widetilde{\mathbf{Q}}_0 = \begin{bmatrix} \mathbf{U}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_0 \end{bmatrix} \mathbf{W}_M \begin{bmatrix} \mathbf{I}_{\lceil M/2 \rceil} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_{\lfloor M/2 \rfloor} \end{bmatrix} \tag{22}$$

where $\mathbf{U}_0$ and $\mathbf{V}_0$ are $\lceil M/2 \rceil \times \lceil M/2 \rceil$ and $\lceil M/2 \rceil \times \lfloor M/2 \rfloor$ nonsingular matrices, respectively. If M is even, the matrices $\mathbf{U}_0$ and $\mathbf{V}_0$ in the above equation obviously have the size $M/2 \times M/2$. Consequently, $\widetilde{\mathbf{Q}}_0$ includes all the solutions for M. The structure is illustrated in Fig. 3. Finally, $\boldsymbol{\Lambda}_{OH}(z)$ is simply implemented as follows:

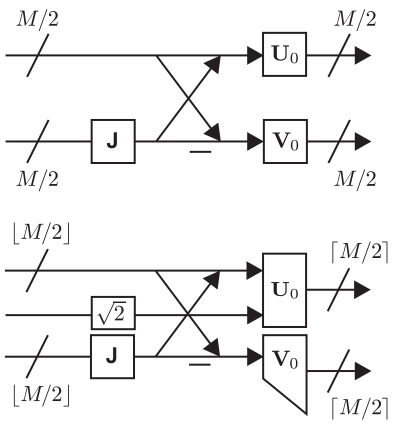

Fig. 3. $\widetilde{\mathbf{Q}}_0$ in Type 1 OLPPRFBs with HOF building blocks: (Top) $\widetilde{\mathbf{Q}}_0$ for even M. (Bottom) $\widetilde{\mathbf{Q}}_0$ for odd M.

$$\boldsymbol{\Lambda}_{OH}(z) = \begin{bmatrix} \hat{\boldsymbol{\Lambda}}_{OH}(z) & \mathbf{0} \\ \mathbf{0} & \hat{\boldsymbol{\Lambda}}'_{OH}(z) \end{bmatrix} \tag{23}$$

where both $\hat{\boldsymbol{\Lambda}}_{OH}(z)$ and $\hat{\boldsymbol{\Lambda}}'_{OH}(z)$ have size $P/2 \times \lceil M/2 \rceil$, in which the powers of z are the same as (16).

In the result, $\widetilde{\mathbf{E}}_0(z)$ in Type 1 OLPPRFBs is represented as follows:

$$\widetilde{\mathbf{E}}_0(z) = \begin{bmatrix} \widetilde{\mathbf{U}}_0 & \mathbf{0} \\ \mathbf{0} & \widetilde{\mathbf{V}}_0 \end{bmatrix} \mathbf{W}_P \boldsymbol{\Lambda}_{OH}(z) \mathbf{W}_{2\lceil M/2 \rceil} \tag{24}$$

where both nonsingular matrices $\widetilde{\mathbf{U}}_0$ and $\widetilde{\mathbf{V}}_0$ are of the size $P/2 \times P/2$ and shown in Fig. 4. The synthesis FB $\mathbf{R}(z)$ can be obtained by the following factorization:

$$\mathbf{R}(z) = \widetilde{\mathbf{Q}}_0^{-1} \mathbf{Q}_1^{-1}(z) \ldots \mathbf{Q}_{K_0}^{-1}(z) \widetilde{\mathbf{E}}_0^{-1}(z) \mathbf{G}_1^{-1}(z) \ldots \mathbf{G}_{K_1}^{-1}(z) \tag{25}$$

where

$$\widetilde{\mathbf{E}}_0^{-1}(z) = \mathbf{W}_{2\lceil M/2 \rceil} \boldsymbol{\Lambda}_{OH}^{-1}(z) \mathbf{W}_P \begin{bmatrix} \widetilde{\mathbf{U}}_0^{-1} & \mathbf{0} \\ \mathbf{0} & \widetilde{\mathbf{V}}_0^{-1} \end{bmatrix} \tag{26}$$

$$\boldsymbol{\Lambda}_{OH}^{-1}(z) = \begin{bmatrix} \hat{\boldsymbol{\Lambda}}_{IOH}(z) & \mathbf{0} \\ \mathbf{0} & \hat{\boldsymbol{\Lambda}}'_{IOH}(z) \end{bmatrix} \tag{27}$$

and both $\hat{\boldsymbol{\Lambda}}_{IOH}(z)$ and $\hat{\boldsymbol{\Lambda}}'_{IOH}(z)$ are $\lceil M/2 \rceil \times P$ matrices guarantee PR. Moreover, the powers of z in $\hat{\boldsymbol{\Lambda}}_{IOH}(z)$ and $\hat{\boldsymbol{\Lambda}}'_{IOH}(z)$ are determined by $s'_{j,i}$ and $s_{j,i}$ in (16), respectively.

3.2. Parameterization of $\hat{\boldsymbol{\Lambda}}_{OH}(1)$: PR case

In the above subsection, the structure of $\widetilde{\mathbf{E}}_0(z)$ for Type 1 OLPPRFBs is derived. The remaining problem is to parameterize a $P/2 \times \lceil M/2 \rceil$ matrix $\hat{\boldsymbol{\Lambda}}_{OH}(1)$ for $\boldsymbol{\Lambda}_{OH}(z)$ which guarantees the PR and order-N conditions. In this paper, we modify the parameterization described in [10]. It is based on the partitioning of the rectangular matrix that a $P \times M$ invertible matrix $\mathbf{A}$ can be always partitioned into an $M \times M$ invertible matrix $\mathbf{A}_0$ and a $(P-M) \times M$ matrix $\mathbf{A}_1$ as follows:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_0 \\ \mathbf{A}_1 \end{bmatrix} \tag{28}$$

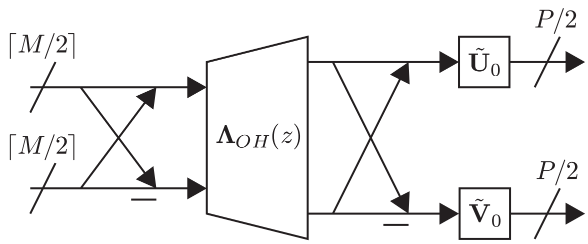

Fig. 4. $\widetilde{\mathbf{E}}_0(z)$ for Type 1 OLPPRFBs.

Furthermore, its left-inverse $\mathbf{A}^{-1}$ is written as

$$\begin{aligned}\mathbf{A}^{-1} &= [\mathbf{A}_0^{-1} - \mathbf{A}_2\mathbf{A}_1\mathbf{A}_0^{-1} \ \ \mathbf{A}_2] \\ &= ([\mathbf{I}_M \ \ \mathbf{0}] + \mathbf{A}_2[-\mathbf{A}_1 \ \ \mathbf{I}]) \begin{bmatrix} \mathbf{A}_0^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{P-M} \end{bmatrix} \end{aligned} \tag{29}$$

where $\mathbf{A}_2$ is an $M \times (P-M)$ arbitrary matrix. The important property in (29) is the existence of freedom in the matrix $\mathbf{A}^{-1}$. It helps to derive the HOF structure.

Based on this factorization, we partition $\hat{\mathbf{\Lambda}}_{OH}(1)$ into an $\lceil M/2 \rceil \times \lceil M/2 \rceil$ invertible matrix $\widetilde{\mathbf{\Lambda}}_0$ and a $(P/2 - \lceil M/2 \rceil) \times \lceil M/2 \rceil$ matrix $\widetilde{\mathbf{\Lambda}}_1$ such as

$$\hat{\mathbf{\Lambda}}_{OH}(1) = \begin{bmatrix} \widetilde{\mathbf{\Lambda}}_0 \\ \widetilde{\mathbf{\Lambda}}_1 \end{bmatrix} \tag{30}$$

Obviously the square invertible matrix $\widetilde{\mathbf{\Lambda}}_0$ is expected to have the similar structure to $\hat{\mathbf{\Lambda}}_{HOF}(1)$ in (14). Therefore, it is written as

$$\widetilde{\mathbf{\Lambda}}_0 = \widetilde{\mathbf{L}}_0 \widetilde{\mathbf{R}}_0 \tag{31}$$

where

$$\widetilde{\mathbf{L}}_0 = \begin{bmatrix} \widetilde{\mathbf{L}}_{0,0} & \mathbf{0} & \\ \mathbf{0} & \ddots & \ddots \\ & \ddots & \widetilde{\mathbf{L}}_{0,n_0} \end{bmatrix}, \quad \widetilde{\mathbf{R}}_0 = \begin{bmatrix} \widetilde{\mathbf{R}}_{0,0} & \mathbf{0} & \\ \mathbf{0} & \ddots & \ddots \\ & \ddots & \widetilde{\mathbf{R}}_{0,n_0} \end{bmatrix} \tag{32}$$

in which $\widetilde{\mathbf{L}}_{0,n}$ and $\widetilde{\mathbf{R}}_{0,n}$ $(n = 0, 1, \ldots)$ are $\tilde{N} \times \tilde{N}$ $(\tilde{N} \leqslant (N+1)/2)$ lower and upper triangular matrices whose diagonal elements are all 1, respectively. Furthermore, $\hat{\mathbf{\Lambda}}_{OH}^{-1}(1)$ can be represented by using (29) and (31) as follows:

$$\begin{aligned}\hat{\mathbf{\Lambda}}_{OH}^{-1}(1) &= ([\mathbf{I}_{\lceil M/2 \rceil} \ \ \mathbf{0}_{\lceil M/2 \rceil \times (P/2 - \lceil M/2 \rceil)}] + \widetilde{\mathbf{\Lambda}}_2[-\widetilde{\mathbf{\Lambda}}_1 \ \ \mathbf{I}_{(P/2-\lceil M/2 \rceil)}]) \\ &\quad \times \begin{bmatrix} \widetilde{\mathbf{R}}_0^{-1}\widetilde{\mathbf{L}}_0^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{(P/2-\lceil M/2 \rceil)} \end{bmatrix}\end{aligned} \tag{33}$$

where $\widetilde{\mathbf{\Lambda}}_2$ is an arbitrary $\lceil M/2 \rceil \times (P/2 - \lceil M/2 \rceil)$ matrix.

Next, we parameterize $\widetilde{\mathbf{\Lambda}}_1$ and $\widetilde{\mathbf{\Lambda}}_2$ to satisfy realizing the order-N HOF building blocks. In this paper, we consider OLPPRFBs with the same filter length in both banks and thus, the left $\lceil M/2 \rceil \times \lceil M/2 \rceil$ submatrix in $\hat{\mathbf{\Lambda}}_{OH}^{-1}(1)$ has to be the block diagonal matrix. It is calculated as $(\mathbf{I} - \widetilde{\mathbf{\Lambda}}_2\widetilde{\mathbf{\Lambda}}_1)\widetilde{\mathbf{R}}_0^{-1}\widetilde{\mathbf{L}}_0^{-1}$. Clearly $\widetilde{\mathbf{R}}_0^{-1}\widetilde{\mathbf{L}}_0^{-1}$ is a block diagonal matrix. Finally, the condition becomes that $\widetilde{\mathbf{\Lambda}}_2\widetilde{\mathbf{\Lambda}}_1$ has to be an arbitrary block diagonal matrix, under the constraint that $\mathbf{\Lambda}_{OH}^{-1}(z)$ does not exceed order-N (Condition 1).

On the other hand, $\widetilde{\mathbf{\Lambda}}_1$ and $\widetilde{\mathbf{\Lambda}}_2$ have to be imposed with the order-N requirements from $s_{i,j}$ and $s'_{i,j}$ in (16) as well as that for $\widetilde{\mathbf{\Lambda}}_0$ (Condition 2). Consequently, the entire condition for $\widetilde{\mathbf{\Lambda}}_1$ and $\widetilde{\mathbf{\Lambda}}_2$ is represented as (Condition 1) ∩ (Condition 2). Based on this, $\widetilde{\mathbf{\Lambda}}_1$ and $\widetilde{\mathbf{\Lambda}}_2$ are finally parameterized as the upper and lower triangular matrices, respectively, shown as follows:

$$\widetilde{\mathbf{\Lambda}}_1 = \begin{bmatrix} 0 & \ldots & 0 & r_{1,1} & r_{1,2} & \ldots & r_{1,n_1} \\ \vdots & \ddots & \vdots & 0 & r_{2,2} & \ldots & r_{2,n_1} \\ \vdots & & \vdots & \vdots & \ddots & \ddots & \vdots \end{bmatrix} \tag{34}$$

$$\widetilde{\mathbf{\Lambda}}_2 = \begin{bmatrix} 0 & \ldots & 0 & l_{1,1} & l_{1,2} & \ldots & l_{1,n_1} \\ \vdots & \ddots & \vdots & 0 & l_{2,2} & \ldots & l_{2,n_1} \\ \vdots & & \vdots & \vdots & \ddots & \ddots & \vdots \end{bmatrix}^T \tag{35}$$

where $r_{m,n}$ and $l_{m,n}$ $(m, n = 1, 2, \ldots)$ are arbitrary coefficients, respectively. Additionally, $n_1 = \min((N-1)/2, s(n_0))$ where $s(n_0)$ is the number of columns of $\mathbf{L}_{0,n_0}$ in (32).

3.3. Parameterization for $\hat{\mathbf{\Lambda}}_{OH}(1)$: PU case

In the PU case, the Condition 1 for order-N PR building blocks described in the previous subsection does not exist since the transposition of an order-N PU analysis building block also has order-N. However, there exists a particular problem in the PU $\hat{\mathbf{\Lambda}}_{OH}(1)$. In (34), $\widetilde{\mathbf{\Lambda}}_1$ has to be an upper triangular matrix from the Condition 2, furthermore, $\widetilde{\mathbf{\Lambda}}_0$ has to be a block diagonal matrix such as (32). After all, this restriction is the same as parameterizing a rectangular PU matrix which is *chipped* at its upper right corner and/or lower left one. For example, if $P = 10$, $M = 6$ and $N = 5$, $\hat{\mathbf{\Lambda}}_{OH}(1)$ has to be

$$\hat{\mathbf{\Lambda}}_{OH}(1) = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \\ 0 & \times & \times \\ 0 & 0 & \times \end{bmatrix} \tag{36}$$

where × denotes any constant and can be ignored from the condition. Since the upper right zero part can be obtained by applying a block diagonal structure, the problem is simplified to parameterize the rectangular matrix chipped at its lower left corner.

In this paper, we consider to parameterize a PU $\hat{\mathbf{\Lambda}}_{OH}(1)$ by modifying the plane rotation of a *filled* tall matrix [21]. The method in [21] is based on removing the first (or last) $(P-M)$ columns from a $P \times P$ square orthogonal matrix. A $P \times P$ orthogonal matrix has $\binom{P}{2}$ rotation angles, whereas $P \times M$ one has $\binom{P}{2} - \binom{P-M}{2}$ rotations. Without loss of generality, we assume to remove the last $(P-M)$ columns from the $P \times P$ square orthogonal matrix $\mathbf{A}$ parameterized by the plane rotation method as follows:

$$\mathbf{A} = \prod_{i=M}^{2} \prod_{j=i-1}^{1} \mathbf{\Theta}_{j,i} \tag{37}$$

$$[\mathbf{\Theta}_{j,i}]_{k,l} = \begin{cases} 1, & k = l \neq i \text{ or } j \\ \cos\theta_{i,j}, & k = l = i \text{ or } j \\ \sin\theta_{i,j}, & k = i \text{ and } l = j \\ -\sin\theta_{i,j}, & k = j \text{ and } l = i \\ 0 & \text{otherwise} \end{cases} \tag{38}$$

If we need the (k,l)-th element in $\mathbf{A}$ located in its lower left corner to be 0, the corresponding $-\sin\theta_{i,j}$ has to be 0. As a result, one off-diagonal element can be 0 by removing the corresponding rotation angle since $\mathbf{\Theta}_{j,i}$ become $\mathbf{I}$ with the condition $\theta_{i,j}=0$. The number of restricted parameters for one chipped element is one, i.e., the constraint is minimal.

4. HOF building blocks for Type 2 OLPPRFBs

This section describes the structure of HOF building block $\widetilde{\mathbf{E}}_0(z)$ for Type 2 OLPPRFBs. First, we assume each $\mathbf{G}_i(z)$ and $\mathbf{Q}_j(z)$ has the same structure as that of the traditional OLPPRFBs. Based on this assumption, a Type 2 OLPPRFB is realized if the simplest structure

$$\mathbf{E}(z)=\widetilde{\mathbf{E}}_0(z)\widetilde{\mathbf{Q}}_0 \tag{39}$$

satisfies the LP and PR conditions since $\mathbf{G}_i(z)$ and $\mathbf{Q}_j(z)$ always propagate these properties. Obviously $\widetilde{\mathbf{Q}}_0$ has order-0, i.e., a constant matrix, thus the structure of $\widetilde{\mathbf{E}}_0(z)$ depends on whether K is even or odd.

4.1. HOF building blocks for Type 2 OLPPRFBs with even K

For even K, the order-N building block $\widetilde{\mathbf{E}}_0(z)$ in (39) has to have odd-order since $K=(N+1)$. It indicates that $\widetilde{\mathbf{E}}_0(z)$ is similar to that of Type 1 OLPPRFBs. In Type 1, there are two oversampling matrices. One is $\widetilde{\mathbf{Q}}_0$, which has the size of $2\lceil M/2\rceil\times M$. The other is $\widetilde{\mathbf{E}}_0(z)$, which is of the size $P\times 2\lceil M/2\rceil$. Finally, we yield a $P\times M$ polyphase matrix for even P. In contrast, P could be both even or odd for Type 2. To be clear the condition about P and M, we discuss the limitation of a rectangular delay matrix $\mathbf{\Lambda}_{OH}(z)$.

We can obtain some patterns for filter symmetry polarity even for the same P and M (see Table 1). However, there is a stricter limitation for HOF building blocks. Similar to (23), the sizes of rectangular matrices $\hat{\mathbf{\Lambda}}_{OH}(z)$ and $\hat{\mathbf{\Lambda}}'_{OH}(z)$ have to be the same to yield LP filters. From this condition, there will exist the restriction $N_s=N_a$ which does not satisfy the requirement, i.e., $N_s\neq N_a$, for Type 2 OLPPRFBs. However, the contradiction can be released by using a rectangular matrix $\mathrm{diag}(\widetilde{\mathbf{U}}_0,\widetilde{\mathbf{V}}_0)$.

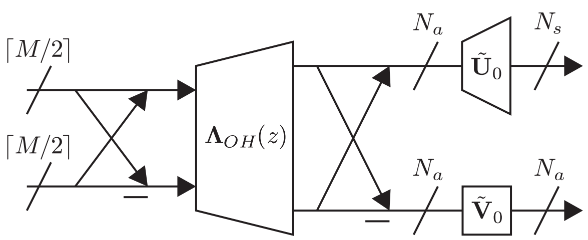

Fig. 5. $\widetilde{\mathbf{E}}_0(z)$ for Type 2 HOF building blocks with even K.

Consequently, the components in (39) is factorized into

$$\widetilde{\mathbf{Q}}_0=\begin{bmatrix}\mathbf{U}_0 & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_0\end{bmatrix}\mathbf{W}_M\begin{bmatrix}\mathbf{I}_{\lceil M/2\rceil} & \mathbf{0}\\ \mathbf{0} & \mathbf{J}_{\lfloor M/2\rfloor}\end{bmatrix} \tag{40}$$

$$\widetilde{\mathbf{E}}_0(z)=\begin{bmatrix}\widetilde{\mathbf{U}}_0 & \mathbf{0}\\ \mathbf{0} & \widetilde{\mathbf{V}}_0\end{bmatrix}\mathbf{W}_{2N_a}\mathbf{\Lambda}_{OH}(z)\mathbf{W}_{2\lceil M/2\rceil} \tag{41}$$

$$\mathbf{\Lambda}_{OH}(z)=\begin{bmatrix}\hat{\mathbf{\Lambda}}_{OH}(z) & \mathbf{0}\\ \mathbf{0} & \hat{\mathbf{\Lambda}}'_{OH}(z)\end{bmatrix} \tag{42}$$

where $\mathbf{U}_0$, $\mathbf{V}_0$, $\widetilde{\mathbf{U}}_0$ and $\widetilde{\mathbf{V}}_0$ are of the sizes $\lceil M/2\rceil\times\lceil M/2\rceil$, $\lceil M/2\rceil\times\lfloor M/2\rfloor$, $N_s\times N_a$ and $N_a\times N_a$, respectively. Moreover, the oversampling delay matrices $\hat{\mathbf{\Lambda}}_{OH}(z)$ and $\hat{\mathbf{\Lambda}}'_{OH}(z)$ are of the size $N_a\times\lceil M/2\rceil$. The lattice structure of $\widetilde{\mathbf{E}}_0(z)$ is implemented in Fig. 5. Obviously, the initial block $\widetilde{\mathbf{Q}}_0$ is the same as that of Type 1.

4.2. HOF building blocks for Type 2 OLPPRFBs with odd K

In contrast to the previous subsection, the odd K case requires an even-ordered $\widetilde{\mathbf{E}}_0(z)$. Unfortunately, the similar structure to Types 1 and 2 for even K cannot be used for this case since all of $\widetilde{\mathbf{E}}_0(z)$ mentioned previously have odd-order. In order to construct $\widetilde{\mathbf{E}}_0(z)$, we apply the odd-channel solution of critically sampled HOF building blocks shown in (20). The critically sampled one is realized by cascading an odd-order HOF building block and an order-1 one, thus in the oversampled extension, the same structure can be considered. Additionally, $\widetilde{\mathbf{Q}}_0$ could be a rectangular matrix to realize the structure for any P and M. Similar to the previous subsection, there is a restriction on $\mathbf{\Lambda}_{OH}(z)$ about N_s and N_a such as $(N_a-1)\leqslant N_s\leqslant(N_a+1)$. However, it can be also relaxed by applying the rectangular matrix $\mathrm{diag}(\widetilde{\mathbf{U}}_0,\widetilde{\mathbf{V}}_0)$. Finally, $\widetilde{\mathbf{E}}_0(z)$ shown in the following realizes a $P\times M$ polyphase matrix:

$$\widetilde{\mathbf{E}}_0(z)=\begin{bmatrix}\widetilde{\mathbf{U}}_1 & \mathbf{0}\\ \mathbf{0} & \widetilde{\mathbf{V}}_1\end{bmatrix}\mathbf{W}_{2N_a+1}\mathbf{\Lambda}_2(z)\mathbf{W}_{2N_a+1}\times\begin{bmatrix}\widetilde{\mathbf{U}}_0 & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{N_a+1}\end{bmatrix}\mathbf{W}_{2N_a+1}\mathbf{\Lambda}_{OH}(z)\mathbf{W}_{2\lfloor M/2\rfloor+1} \tag{43}$$

where $\widetilde{\mathbf{U}}_0$, $\widetilde{\mathbf{U}}_1$ and $\widetilde{\mathbf{V}}_1$ are $N_a\times N_a$, $N_s\times N_a$ and $N_a\times N_a$ invertible matrices, respectively, and $\mathbf{\Lambda}_2(z)=\mathrm{diag}(z^{-1}\mathbf{I}_{N_a},\mathbf{I}_{N_a+1})$. $\mathbf{\Lambda}_{OH}(z)$ clearly has the size of $(2N_a+1)\times(2\lfloor M/2\rfloor+1)$, and takes the form of

$$\mathbf{\Lambda}_{OH}(z)=\begin{bmatrix}\hat{\mathbf{\Lambda}}_{OH}(z) & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & z^{-(N-1)/2} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \hat{\mathbf{\Lambda}}'_{OH}(z)\end{bmatrix} \tag{44}$$

where both $\hat{\mathbf{\Lambda}}_{OH}(z)$ and $\hat{\mathbf{\Lambda}}'_{OH}(z)$ have the size of $N_a\times\lfloor M/2\rfloor$. The structure is shown in Fig. 6. Additionally, $\widetilde{\mathbf{Q}}_0$ is the same as (40) but the sizes of $\mathbf{U}_0$ and $\mathbf{V}_0$ are $(\lfloor M/2\rfloor+1)\times\lceil M/2\rceil$ and $\lfloor M/2\rfloor\times\lfloor M/2\rfloor$, respectively, and is represented in Fig. 7.

Finally, Type 2 OLPPRFBs with HOF building blocks is regarded as an alternative structure of Type 2 traditional lattice structure since any P, M, N_s and N_a are permitted by using the structure described in this section.

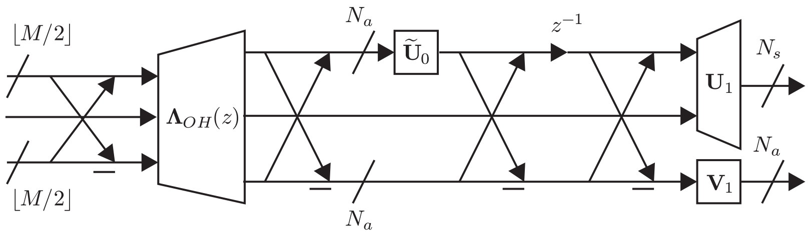

Fig. 6. $\widetilde{\mathbf{E}}_0(z)$ for Type 2 HOF building blocks with odd K.

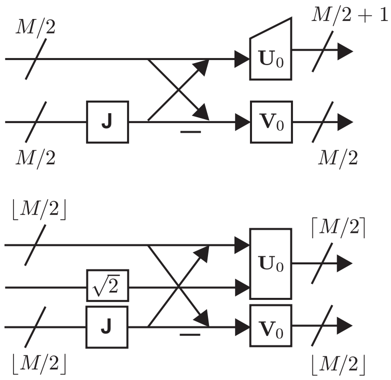

Fig. 7. $\widetilde{\mathbf{Q}}_0$ in Type 2 OLPPRFBs with HOF building blocks for odd K: (Top) $\widetilde{\mathbf{Q}}_0$ for even M. (Bottom) $\widetilde{\mathbf{Q}}_0$ for odd M.

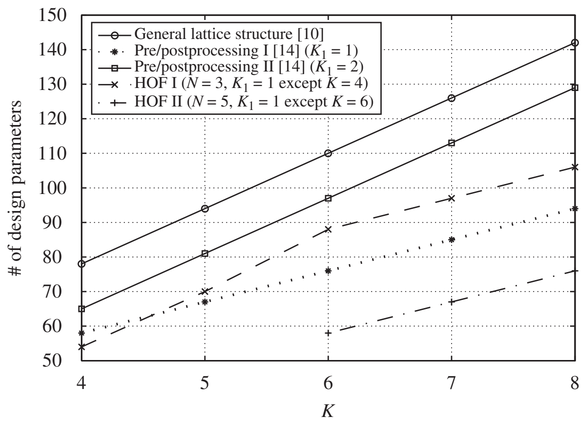

Fig. 8. The comparison of the design parameters for some K.

5. The number of design parameters and design examples

5.1. Comparison of the number of design parameters

In order to validate our proposed structure, this section describes the comparison of the performance to other traditional lattice structures for OLPPRFBs. First, the number of design parameters is compared. In this section, the comparison for only Type 1/even M case is presented since it can avoid cumbersome classifications based on P, M, K, N_s and N_a, and [14] was only considered in that case. The comparison is shown in Fig. 8 for $P = 8$ and $M = 6$. In the figure, we compare the number of design parameters as functions of K. The OLPPRFBs considered are listed as follows:

- General lattice structure [10].
- Pre- and postprocessing structure [14] I: $K_1 = 1$:
- Pre- and postprocessing structure [14] II: $K_1 = 2$:
- HOF structure I: $N = 3$ and $K_1 = 1$ except $K = 4$:
- HOF structure II: $N = 5$ and $K_1 = 1$ except $K = 6$.

For the general lattice structure and HOF ones, the design parameters existing in the synthesis banks are included. In the pre- and postprocessing system, there are some freedom in the choice of K_0 and K_1 corresponding to the number of pre- and postprocessing building blocks, respectively. Therefore two choices are implemented. Furthermore, in the HOF structure, there exists a freedom about N which is the order of $\widetilde{\mathbf{E}}_0(z)$. We show two cases of $N = 3$ and 5. In the comparison, K_1 should be 0 for the least order cases of the HOF structures since they do not need $\mathbf{G}_1(z)$ to realize the desired filter lengths.

Obviously, the general lattice structure always has the most number of design parameters and the second is the pre- and postprocessing system II. The remaining three are below these OLPPRFBs. The HOF structure I has almost the same number of design parameters as the pre- and postprocessing I from $K = 4$ to 5, whereas it has slightly more design parameters in the other K. The HOF structure II has totally fewer parameters than the others. From this figure, one can see that the HOF structure provides more flexibility than the traditional approaches which depends on N, K_0 and K_1.

5.2. Design examples

Next, we show design examples of OLPPRFBs with proposed HOF building blocks based on the structures in Sections 3 and 4. All examples have been optimized with stopband attenuation formulated as follows:

$$C_{stopband} = \sum_{i=0}^{M-1} \int_{\omega \in \Omega_{stopband}} W_a(\omega)|H_i(e^{j\omega})|^2\, d\omega + \sum_{i=0}^{M-1} \int_{\omega \in \Omega_{stopband}} W_s(\omega)|F_i(e^{j\omega})|^2\, d\omega \tag{45}$$

where $W_a(\omega)$ and $W_s(\omega)$ are weighting functions for the analysis and synthesis banks, respectively. In this paper, we set $W_a(\omega) = 1$ and $W_s(\omega) = 1$ for all examples. The

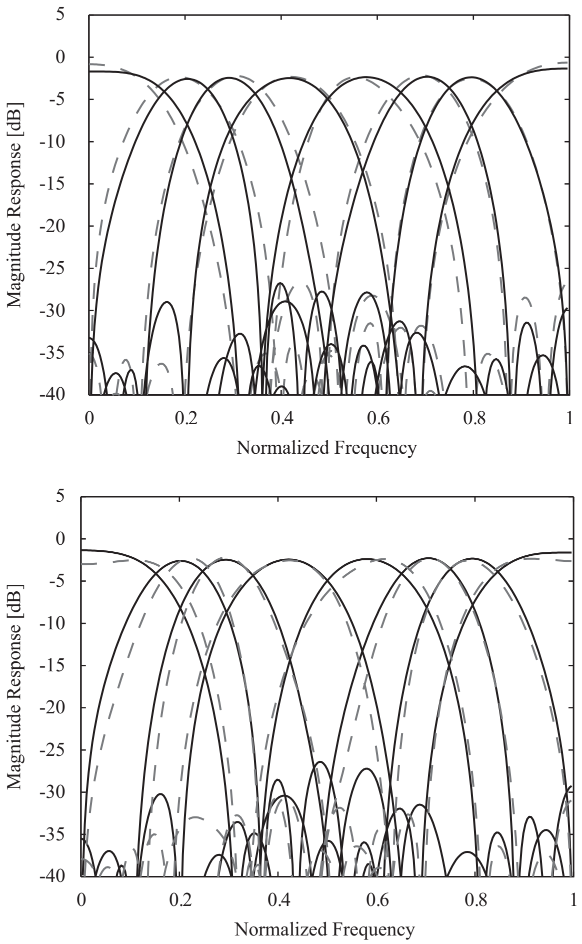

Fig. 9. Comparison between Design example 1 of Type 1 OLPPRFB with a HOF building block (solid line) and pre- and postprocessing system II [14] (dashed line), both have $P=8$, $M=6$ and $L=36$. (Top) Analysis bank. (Bottom) Synthesis bank.

Table 2
Comparison of stopband attenuations for OLPPRFBs with $P=8$, $M=6$ and $L=36$.

Average stopband attenuation (-dB)	Pre-/postprocessing system II [14]	HOF structure
Analysis bank	42.80	41.44
Synthesis bank	42.54	41.56

design examples are listed as follows:

Design example 1: Type 1 PR solution for $P=8$, $M=6$, $K_0=K_1=1$, and $N=3$ ($L=36$).

This example corresponds to Section 3. The frequency responses of both banks are shown as solid lines in Fig. 9. The responses of pre- and postprocessing system [14] II which has the same filter length 36 are depicted as dashed lines in the figure for the comparison of filter characteristics. The objective comparison of filter performance is shown in Table 2. In both banks, these two filter banks show very comparable results, whereas the number of design parameters in the proposed system is 79, which is 11% fewer than 89 of the pre- and postprocessing system II.

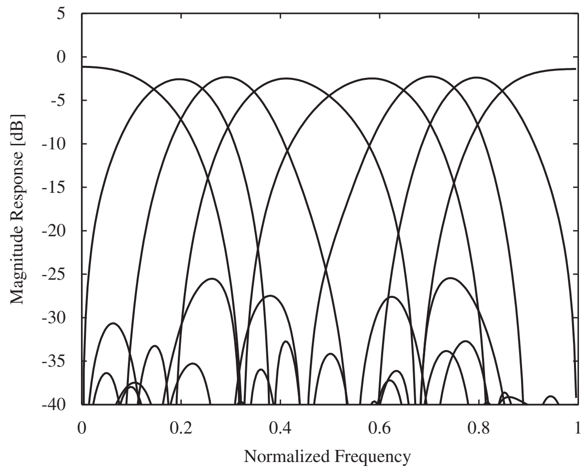

Fig. 10. Design example 2. Type 1 OLPPUFB with a HOF building block.

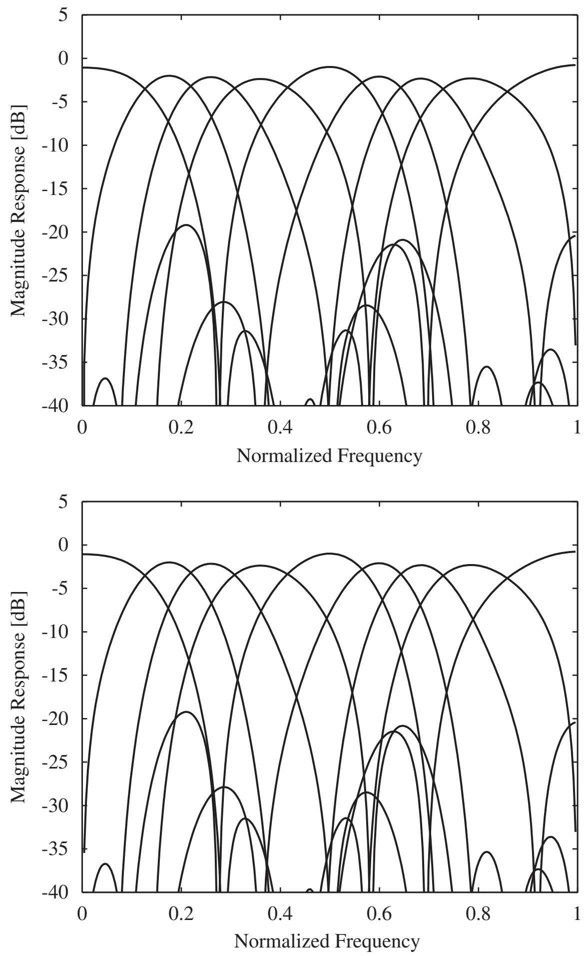

Fig. 11. Design example 3. Type 2 OLPPRFB with a HOF building block. (Top) Analysis bank. (Bottom) Synthesis bank.

Design example 2: Type 1 PU solution for $P = 8$, $M = 6$, $K_0 = 0$, $K_1 = 1$, and $N = 5$ ($L = 42$).

This OLPPUFB is parameterized by using the method in Section 3. The analysis frequency response is implemented in Fig. 10. The number of rotation angles (free design parameters) is 29.

Design example 3: Type 2 PR solution for $P = 9$, $M = 7$, $K_0 = K_1 = 0$, and $N = 3$ ($L = 35$).

Its frequency responses are shown in Fig. 11. This is an example of Type 2 OLPPRFBs described in Section 4.

Obviously, all of these examples present good filter characteristics. Similar to the traditional one [10], there are some design freedoms corresponding to *only* the synthesis bank in the PR solution. It implies that we can design multiple synthesis banks which have slightly different filter characteristics corresponding to one analysis bank, if needed.

6. Conclusions

In this paper, we presented a new lattice component of OLPPRFBs called HOF building block. OLPPRFBs with HOF building blocks have following desirable properties:

(1) They have fewer number of design parameters:
(2) They have fewer number of building blocks:
(3) They include all possible solutions for P, M, K, N_s and N_a:
(4) The PU structure can be factorized.

Additionally, parameterizations of rectangular matrices for the HOF structure are provided. These indicate that the proposed OLPPRFB is an alternative structure of the traditional ones while maintaining their merits. The proposed building block is also regarded as an extension of the critically sampled HOF structure, hence HOF building blocks cover all the patterns for $P \geqslant M$. Compared to the traditional successful approaches to factorize the polyphase matrices of FBs into order-1 building blocks, factorizing FBs into order-N is an attractive alternative system in practical signal processing due to small implementation costs.

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