Oversampled Linear-Phase Perfect Reconstruction Filter Banks with Higher-Order Feasible Building Blocks: Structure and Parameterization

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***Abstract*— This paper proposes new building blocks for the lattice structure of oversampled linear-phase perfect reconstruction filter banks (OLPPRFBs). The structure is an extended version of higher-order feasible building blocks for critically-sampled LPPRFBs. It uses fewer number of building blocks and design parameters than those of traditional OLPPRFBs, whereas the frequency characteristic of the new OLPPRFB is comparable to that of traditional one.**

I. Introduction

Oversampled filter bank (OFB) is one of the most developing issues in the field of FB theory [1]–[3]. OFB is a superclass of critically-sampled FBs since its polyphase matrix has the size $P \times M$ ($P > M$) which covers entire $M \times M$ classes for critically-sampled ones. They have more design freedom than the critically-sampled FBs, thus they are expected to have good frequency characteristics and/or robustness against errors. Additionally, linear-phase (LP) and perfect reconstruction (PR) properties are usually highly desired in practical signal processing framework [4], [5]. In this paper, we consider to derive a new structure of OLPPRFBs.

Gan *et al.* investigated lattice structures for OLPPRFBs and proved the necessary conditions which are regarded as the extended version of those for critically-sampled LPPRFBs [2]. However, their structure has a $P \times M$ rectangular starting lattice block. Following the block, critically-sampled P-channel building blocks are cascaded to yield desired filter length $L = KM$ in spite of the downsampling factor M. Hence, the calculation cost is much higher than the critically-sampled M-channel ones. Liang *et al.* proposed a subset of Gan's structure which is pre- and postprocessing system of a $P \times M$ oversampling matrix [6]. The alternative structure helps to decrease the calculation cost for long filters. However, most of the previous works on the lattice structure of OLPPRFBs are based on cascading order-1 building blocks [2], [6]. It is effective in the viewpoint of achieving any-order FBs. Unfortunately, the high-order FB requires many order-1 building blocks which increase the implementation cost.

The authors proposed a lattice component of critically-sampled biorthogonal (BO) LPFBs [7] called higher-order feasible (HOF) building block. It realizes an order-N ($N \geq 2$) LPPRFB with one HOF building block. Additionally, it is also regarded as an extended class of traditional (critically-sampled) order-1 building blocks. The advantage of HOF building blocks against cascaded order-1 ones is its ability to realize comparable performance with fewer number of building blocks and design parameters for the same filter lengths.

We extend the HOF building block from critically-sampled to oversampled LPPRFBs. It inherits all the merits on critically-sampled HOF structure, furthermore, it has an attractive feature of a $P \times M$ rectangular oversampling matrix which exists particularly on the HOF structure. The parameterization which is a specific method for the HOF building blocks is described in the paper. In the design example, our OLPPRFB based on HOF buiding blocks shows the comparable frequency characteristic to that based on cascaded order-1 structure in spite of fewer number of design parameters.

Remaining parts of this paper are organized as follows: Section II reviews traditional structures of OLPPRFBs and critically-sampled HOF building blocks. Section III describes the oversampled HOF structure. Section IV validates our proposed structure by comparing with the traditional one. Finally, the paper concludes in Section V.

Notations: $\lceil \cdot \rceil$ and $\lfloor \cdot \rfloor$ define the functions which return ceiling and floor of $(\cdot)$, respectively. We also define the special matrices of size $S \times S$; $\mathbf{I}_S$ is the identity matrix, $\mathbf{J}_S$ is the reversal identity matrix and $\mathbf{0}_S$ is the null matrix. The sizes will be omitted if they are obvious.

II. Review

A. General Lattice Structure of OLPPRFBs

Consider a P-channel OLPPRFB with its filter length $L = KM$ (denoted $P \times KM$, hereafter), N_s symmetric filters, N_a antisymmetric ones, and a downsampling factor of M ($P > M$). The lattice structure is represented as follows [2]:

$$\mathbf{E}(z) = \mathbf{G}_{K-1}(z)\mathbf{G}_{K-2}(z)\ldots\mathbf{G}_1(z)\mathbf{E}_0(z) \tag{1}$$

where $\mathbf{G}_i(z)$ ($i = 1, 2, \ldots, K-1$) and $\mathbf{E}_0(z)$ have the sizes of $P \times P$ and $P \times M$, respectively. If PR is achieved, the causal synthesis polyphase matrix $\mathbf{R}(z)$ is given as

$$\mathbf{R}(z) = z^{-(K-1)}\mathbf{E}_0^{-1}(z)\mathbf{G}_1^{-1}(z)\mathbf{G}_2^{-1}(z)\ldots\mathbf{G}_{K-1}^{-1}(z) \tag{2}$$

where $\mathbf{E}_0^{-1}(z)$ is an $M \times P$ left-inverse matrix of $\mathbf{E}_0(z)$. We also introduce the types of OLPPRFBs described in [2]:

- Type 1 OLPPRFBs: $N_s = N_a$, the order of $\mathbf{G}_i(z) = 1$.
- Type 2 OLPPRFBs: $N_s \neq N_a$, the order of $\mathbf{G}_i(z) \neq 1$.

Note that all of odd-channel OLPPRFBs are categorized in Type 2. In contrast, the even-channel ones can be both types. For the simplicity purpose and limitation of space, we consider the Type 1 structure hereafter. The lattice components in P-channel Type 1 OLPPRFBs are formulated as follows:

$$\mathbf{G}_i(z) = \mathrm{diag}(\mathbf{U}_i, \mathbf{I}_{P/2})\mathbf{W}_P\boldsymbol{\Lambda}(z)\mathbf{W}_P \tag{3}$$

$$\mathbf{E}_0(z) = \mathrm{diag}(\mathbf{U}_0, \mathbf{V}_0)\mathbf{W}_M\mathrm{diag}(\mathbf{I}_{\lceil M/2\rceil}, \mathbf{J}_{\lfloor M/2\rfloor}) \tag{4}$$

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where

$$\mathbf{W}_S = \begin{cases} \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I}_s & \mathbf{I}_s \\ \mathbf{I}_s & -\mathbf{I}_s \end{bmatrix} & S = 2s \\ \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I}_s & \mathbf{0} & \mathbf{I}_s \\ \mathbf{0} & \sqrt{2} & \mathbf{0} \\ \mathbf{I}_s & \mathbf{0} & -\mathbf{I}_s \end{bmatrix} & S = 2s+1 \end{cases}$$

$$\mathbf{\Lambda}(z) = \text{diag}(z^{-1}\mathbf{I}_{P/2}, \mathbf{I}_{P/2}).$$

The size of $\mathbf{U}_i$ is $P/2 \times P/2$, and those of $\mathbf{U}_0$ and $\mathbf{V}_0$ are $P/2 \times \lceil M/2 \rceil$ and $P/2 \times \lfloor M/2 \rfloor$, respectively. The condition that all the matrices have the left-inverse yields the general OLPPRFBs. In this case, the rectangular matrices $\mathbf{U}_0$ and $\mathbf{V}_0$ are often parameterized with Moore-Penrose pseudo inverse [3] or the LDU decomposition [2]. The pseudo inverse method has an advantage that it provides the uniqueness of the inverse, whereas the LDU decomposition provides the possibility to increase the design freedom which belongs to only the synthesis bank (in $\mathbf{U}_0^{-1}$ and $\mathbf{V}_0^{-1}$).

B. OLPPRFBs with Pre- and Postprocessing

In [6], a subset in the general lattice structure of Type 1 OLPPRFBs was proposed. It can be realized to adopt a critically-sampled M-channel LPPRFB by applying to a general P-channel OLPPRFB as a preprocessing system. The lattice structure for Type 1 OLPPRFBs with even M has the form

$$\begin{aligned} \mathbf{E}(z) = &\mathbf{G}_{K_1}(z)\mathbf{G}_{K_1-1}(z)\dots\mathbf{G}_1(z)\bar{\mathbf{E}}_0(z) \\ &\times \mathbf{Q}_{K_0}(z)\mathbf{Q}_{K_0-1}(z)\dots\mathbf{Q}_1(z)\mathbf{Q}_0 \end{aligned} \quad (5)$$

where $K_0 + K_1 = K - 1$ and

$$\bar{\mathbf{E}}_0(z) = \text{diag}(\bar{\mathbf{U}}_0, \bar{\mathbf{V}}_0)\mathbf{W}_M\mathbf{\Lambda}(z)\mathbf{W}_M \quad (6)$$

$$\mathbf{Q}_j(z) = \text{diag}(\mathbf{U}_j, \mathbf{I}_{M/2})\mathbf{W}_M\mathbf{\Lambda}(z)\mathbf{W}_M \quad (7)$$

$$\mathbf{Q}_0 = \text{diag}(\mathbf{U}_0, \mathbf{I}_{M/2})\mathbf{W}_M\text{diag}(\mathbf{I}_{M/2}, \mathbf{J}_{M/2}). \quad (8)$$

In the above equations, $\bar{\mathbf{U}}_0$, $\bar{\mathbf{V}}_0$ and $\mathbf{U}_j$ are $P/2 \times M/2$, $P/2 \times M/2$ and $M/2 \times M/2$ nonsingular matrices, respectively. The structure can be implemented as a pre- and postprocessing system of an oversampling building block $\bar{\mathbf{E}}_0(z)$. Obviously, it does not cover all classes of the general OLPPRFBs shown in the previous subsection, however, it requires fewer number of delay elements and design parameters than the general ones. This is a trade-off between design cost and filter performance. In the case that we need the long filter length, fewer design parameters are much recommended than the slightly modification of the filter characteristics. We consider a new lattice structure of OLPPRFBs based on the pre- and postprocessing system.

C. Critically-Sampled BOLPFBs with HOF Building Blocks

In [7], a lattice structure of critically-sampled BOLPFBs which realize various filter length patterns with one building block was proposed. The order-N building block structure for even-channel BOLPFBs is represented as follows:

$$\mathbf{G}^e_{HOF}(z) = \text{diag}(\mathbf{U}, \mathbf{V})\mathbf{W}_M\mathbf{\Lambda}^e_{HOF}(z)\mathbf{W}_M \quad (9)$$

where the $\mathbf{U}$ and $\mathbf{V}$ are $M/2 \times M/2$ nonsingular matrices, respectively, and

$$\mathbf{\Lambda}^e_{HOF}(z) = \text{diag}(\hat{\mathbf{\Lambda}}_{HOF}(z_{i,j}), \hat{\mathbf{\Lambda}}_{HOF}(z'_{i,j})). \quad (10)$$

Besides, $\hat{\mathbf{\Lambda}}_{HOF}(z_{i,j})$ is a $M/2 \times M/2$ nonsingular matrix included some delay elements. Moreover, $\hat{\mathbf{\Lambda}}_{HOF}(z'_{i,j})$ is the same as $\hat{\mathbf{\Lambda}}_{HOF}(z_{i,j})$ except powers of z. In (10),

$$\hat{\mathbf{\Lambda}}_{HOF}(1) = \mathbf{L}_h\mathbf{R}_h \quad (11)$$

$$\mathbf{L}_h = \text{diag}(\mathbf{L}_{h,0}, \mathbf{L}_{h,1}, \dots),\ \mathbf{R}_h = \text{diag}(\mathbf{R}_{h,0}, \mathbf{R}_{h,1}, \dots) \quad (12)$$

where $\mathbf{L}_{h,n}$ and $\mathbf{R}_{h,n}$ $(n = 0, 1, \dots)$ are $\tilde{N} \times \tilde{N}$ $(\tilde{N} \le \frac{N+1}{2})$ lower and upper triangular matrices whose all diagonal elements are 1, respectively. Moreover, we assume the size of $\mathbf{L}_{h,0}$ and $\mathbf{R}_{h,0}$ is $\frac{N+1}{2} \times \frac{N+1}{2}$ for simplicity. Furthermore, the power of the (i, j)-th element in $\hat{\mathbf{\Lambda}}_{HOF}(z_{i,j})$ and $\hat{\mathbf{\Lambda}}_{HOF}(z'_{i,j})$ is defined by $s_{i,j}$ and $s'_{i,j}$, respectively, where

$$s_{i,j} = \begin{cases} s_{i,i} = \frac{-(N+1)}{2} \\ s_{i,j+1} = s_{i,j} + 1 \\ s_{i+1,j} = s_{i,j} - 1 \end{cases}, \qquad s'_{i,j} = s_{j,i} + 1. \quad (13)$$

Its inverse for the synthesis bank is also represented as

$$\mathbf{G}^{e^{-1}}_{HOF}(z) = \mathbf{W}_M\mathbf{\Lambda}^e_{IHOF}(z)\mathbf{W}_M\text{diag}(\mathbf{U}^{-1}, \mathbf{V}^{-1}). \quad (14)$$

$\mathbf{\Lambda}^e_{IHOF}(z)$ is formulated as

$$\mathbf{\Lambda}^e_{IHOF}(z) = \text{diag}(\hat{\mathbf{\Lambda}}^e_{IHOF}(z'_{j,i}), \hat{\mathbf{\Lambda}}^e_{IHOF}(z_{j,i})) \quad (15)$$

where $\hat{\mathbf{\Lambda}}^e_{IHOF}(1) = \mathbf{R}_h^{-1}\mathbf{L}_h^{-1}$.

III. Oversampled HOF Building Blocks

In this section, we describe the extension of the HOF building blocks from the critically-sampled case to the oversampled one for Type 1 OLPPRFBs. First of all, we assume new OLPPRFBs are factorized as follows:

$$\begin{aligned} \mathbf{E}(z) = &\mathbf{G}_{K_1}(z)\mathbf{G}_{K_1-1}(z)\dots\mathbf{G}_1(z)\widetilde{\mathbf{E}}_0(z) \\ &\times \mathbf{Q}_{K_0}(z)\mathbf{Q}_{K_0-1}(z)\dots\mathbf{Q}_1(z)\widetilde{\mathbf{Q}}_0, \end{aligned} \quad (16)$$

where $\widetilde{\mathbf{E}}_0(z)$ is an oversampling building block and $\widetilde{\mathbf{Q}}_0$ is supposed to be the similar structure to (8).

Second, to apply the HOF structure to OLPPRFBs, we categorize further the structure in (16) as follows:

- Type x.1: A HOF building block is applied at $\mathbf{Q}_j(z)$.
- Type x.2: A HOF building block is applied at $\mathbf{G}_i(z)$.
- Type x.3: A HOF building block is applied at $\widetilde{\mathbf{E}}_0(z)$,

where x denotes 1 or 2. Clearly the Types x.1 and x.2 are the same as a critically-sampled HOF building block in (9), thus we consider the Type x.3 hereafter. In the proposed structure, we consider OLPPRFBs with the same filter length filters in both analysis and synthesis banks.

A. Type x.3 OLPPRFBs with HOF Building Blocks

To factorize $\widetilde{\mathbf{E}}_0(z)$ for the Type x.3, initially the condition for P and M has to be considered. With the assumption that $\widetilde{\mathbf{E}}_0(z)$ is factorized into the similar structure to (6) shown as

$$\widetilde{\mathbf{E}}_0(z) = \text{diag}(\widetilde{\mathbf{U}}_0, \widetilde{\mathbf{V}}_0)\mathbf{W}\mathbf{\Lambda}_{OH}(z)\mathbf{W},$$

there could be two patterns for an oversampling matrix such as

- Pattern 1: oversampling at the block diagonal matrix $\text{diag}(\widetilde{\mathbf{U}}_0, \widetilde{\mathbf{V}}_0)$
- Pattern 2: oversampling at the delay matrix $\mathbf{\Lambda}_{OH}(z)$.

It is obvious that the Pattern 2 is totally different from the other OLPPRFBs, thus, we consider the structure of $\mathbf{\Lambda}_{OH}(z)$.

First, the size of $\mathbf{\Lambda}_{OH}(z)$ is assumed to be $P \times M$ as the simplest approach. However, note that M would be even or odd and $P \times KM$ Type 1 OLPPRFBs could have any K [2]. If we need one for odd M, the number of columns of $\mathbf{\Lambda}_{OH}(z)$ has to be odd. Unfortunately, the problem that only one $\mathbf{\Lambda}_{OH}(z)$ which has the odd number of columns can not yield LP filters with the same length is arised. It prevents the resulting FB from realizing any K solution, and is similar to the odd-channel critically-sampled LPPRFBs [8]. To avoid the disadvantage,

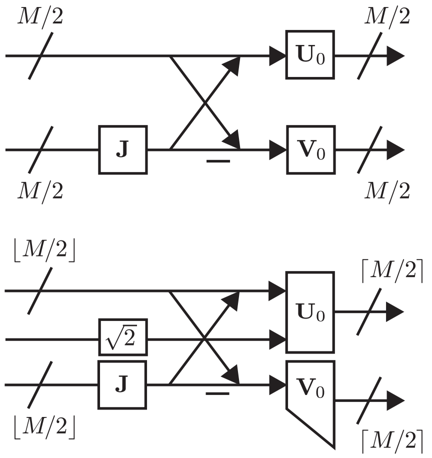

Fig. 1. $\widetilde{\mathbf{Q}}_0$ for Type 1.3 OLPPRFBs: (Top) $\widetilde{\mathbf{Q}}_0$ for even M. (Bottom) $\widetilde{\mathbf{Q}}_0$ for odd M.

we consider to extend $\mathbf{\Lambda}^e_{HOF}(z)$ in (10) to the Type x.3 OLPPRFBs. For odd M, the structure has to be slightly modified. As previously mentioned, we assume $\mathbf{\Lambda}_{OH}(z)$ has the even number of columns, thus it should have the size $P \times (M+1)$ for odd M. To adjust the number of columns for that case and realize any-order FBs, we assume that the initial block is a matrix of the size $2\lceil M/2 \rceil \times M$ represented as

$$\widetilde{\mathbf{Q}}_0 = \text{diag}(\mathbf{U}_0, \mathbf{V}_0)\mathbf{W}_M \text{diag}(\mathbf{I}_{\lceil M/2 \rceil}, \mathbf{J}_{\lfloor M/2 \rfloor}) \tag{17}$$

where $\mathbf{U}_0$ and $\mathbf{V}_0$ are $\lceil M/2 \rceil \times \lceil M/2 \rceil$ and $\lceil M/2 \rceil \times \lfloor M/2 \rfloor$ non-singular matrices, respectively. If M is even, the $\mathbf{U}_0$ and $\mathbf{V}_0$ in the above equation are obviously of the size $M/2 \times M/2$. Consequently, this $\widetilde{\mathbf{Q}}_0$ includes all the solution for M. The structure is depicted in Fig. 1. With this assumption, $\mathbf{\Lambda}_{OH}(z)$ can be implemented as follows:

$$\mathbf{\Lambda}_{OH}(z) = \text{diag}(\hat{\mathbf{\Lambda}}_{OH}(z_{i,j}), \hat{\mathbf{\Lambda}}_{OH}(z'_{i,j})) \tag{18}$$

where both $\hat{\mathbf{\Lambda}}_{OH}(z_{i,j})$ and $\hat{\mathbf{\Lambda}}_{OH}(z'_{i,j})$ have the size $P/2 \times \lceil M/2 \rceil$, in which $z_{i,j}$ and $z'_{i,j}$ are defined with the same powers as (13). In (18), the sizes of $\hat{\mathbf{\Lambda}}_{OH}(z_{i,j})$ and $\hat{\mathbf{\Lambda}}_{OH}(z'_{i,j})$ have to be the same since $\mathbf{\Lambda}_{OH}(z)$ is expected to expand the filter length symmetrically for LP filters. If they have different sizes from each other, the resulting FB can not guarantee the LP property.

In the result, $\widetilde{\mathbf{E}}_0(z)$ of the Type x.3 OLPPRFBs are represented as follows:

$$\widetilde{\mathbf{E}}_0(z) = \text{diag}(\widetilde{\mathbf{U}}_0, \widetilde{\mathbf{V}}_0)\mathbf{W}_P \mathbf{\Lambda}_{OH}(z)\mathbf{W}_{2\lceil M/2 \rceil}. \tag{19}$$

Furthermore, both nonsingular matrices $\widetilde{\mathbf{U}}_0$ and $\widetilde{\mathbf{V}}_0$ are of the size $P/2 \times P/2$. The synthesis FB $\mathbf{R}(z)$ can be obtained in the following factorization:

$$\begin{aligned}\mathbf{R}(z) =& \widetilde{\mathbf{Q}}_0^{-1}\mathbf{Q}_1^{-1}(z)\dots\mathbf{Q}_{K_0}^{-1}(z)\widetilde{\mathbf{E}}_0^{-1}(z) \\ & \times \mathbf{G}_1^{-1}(z)\mathbf{G}_2^{-1}(z)\dots\mathbf{G}_{K_1}^{-1}(z)\end{aligned} \tag{20}$$

where

$$\widetilde{\mathbf{E}}_0^{-1}(z) = \mathbf{W}_{2\lceil M/2 \rceil}\mathbf{\Lambda}_{OH}^{-1}(z)\mathbf{W}_P \text{diag}(\widetilde{\mathbf{U}}_0^{-1}, \widetilde{\mathbf{V}}_0^{-1}) \tag{21}$$

$$\mathbf{\Lambda}_{OH}^{-1}(z) = \text{diag}(\hat{\mathbf{\Lambda}}_{IOH}(z'_{j,i}), \hat{\mathbf{\Lambda}}_{IOH}(z_{j,i})) \tag{22}$$

where $\hat{\mathbf{\Lambda}}_{IOH}(1)$ is a left-inverse of $\hat{\mathbf{\Lambda}}_{OH}(1)$. $\widetilde{\mathbf{E}}_0(z)$ is represented in Fig. 2.

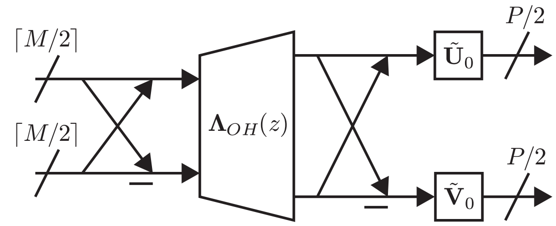

Fig. 2. $\widetilde{\mathbf{E}}_0(z)$ for Type 1.3 OLPPRFBs.

B. Parameterization of $\hat{\mathbf{\Lambda}}_{OH}(1)$

In the above subsection, the structure of $\widetilde{\mathbf{E}}_0(z)$ for the Type 1.3 OLPPRFBs is derived. the remaining problem is to parameterize a $P/2 \times \lceil M/2 \rceil$ left-invertible matrix $\hat{\mathbf{\Lambda}}_{OH}(1)$ in $\mathbf{\Lambda}_{OH}(z)$. In an easily understood manner, $\hat{\mathbf{\Lambda}}_{OH}(1)$ is divided into an $\lceil M/2 \rceil \times \lceil M/2 \rceil$ invertible matrix $\hat{\mathbf{\Lambda}}_{OH,0}$ and a $(P/2 - \lceil M/2 \rceil) \times \lceil M/2 \rceil$ matrix $\hat{\mathbf{\Lambda}}_{OH,1}$ [2] such as

$$\hat{\mathbf{\Lambda}}_{OH}(1) = \begin{bmatrix} \hat{\mathbf{\Lambda}}_{OH,0} \\ \hat{\mathbf{\Lambda}}_{OH,1} \end{bmatrix}. \tag{23}$$

If $\mathbf{\Lambda}_{OH}(z)$ has order-N, $\hat{\mathbf{\Lambda}}_{OH,1}$ has to be an upper triangular matrix represented as

$$\hat{\mathbf{\Lambda}}_{OH,1} = \begin{bmatrix} 0 & \dots & 0 & r_{0,0} & r_{0,1} & \dots & r_{0,(N-1)/2-1} \\ \vdots & \ddots & \vdots & 0 & r_{1,1} & \dots & r_{1,(N-1)/2-1} \\ \vdots & & \vdots & \vdots & \ddots & \ddots & \vdots \end{bmatrix}. \tag{24}$$

Furthermore, $\hat{\mathbf{\Lambda}}_{OH}(1)$ is expected to have similar structure to $\hat{\mathbf{\Lambda}}_{HOF}(1)$ in (11). Hence, the entire decomposition of $\hat{\mathbf{\Lambda}}_{OH}(1)$ is obtained by using the LDU decomposition as follows:

$$\hat{\mathbf{\Lambda}}_{OH}(1) = \begin{bmatrix} \mathbf{L}_{OH,0}\mathbf{R}_{OH,0} \\ \mathbf{R}_{OH,1} \end{bmatrix} \tag{25}$$

where $\mathbf{L}_{OH,0}$ and $\mathbf{R}_{OH,0}$ are the same structures as these in the critically-sampled $\mathbf{L}_h$ and $\mathbf{R}_h$ such as

$$\begin{aligned}\mathbf{L}_{OH,0} &= \text{diag}(\mathbf{L}_{OH,0}^{(0)}, \mathbf{L}_{OH,0}^{(1)}, \dots) \\ \mathbf{R}_{OH,0} &= \text{diag}(\mathbf{R}_{OH,0}^{(0)}, \mathbf{R}_{OH,0}^{(1)}, \dots)\end{aligned} \tag{26}$$

in which $\mathbf{L}_{OH,0}^{(n)}$ and $\mathbf{R}_{OH,0}^{(n)}$ $(n = 0, 1, \dots)$ are $\tilde{N} \times \tilde{N}$ $(\tilde{N} \leq \frac{N+1}{2})$ lower and upper triangular matrices whose all diagonal elements are 1, respectively. Furthermore, $\mathbf{R}_{OH,1} = \hat{\mathbf{\Lambda}}_{OH,1}$.

The next problem is to guarantee that the filter length of the synthesis bank is the same as that in the analysis one. Generally, the left-inverse of a rectangular matrix can be obtained by using Moore-Penrose pseudo inverse [9]. An arbitrary $P \times M$ tall matrix $\mathbf{A}$ has the unique $M \times P$ pseudo inverse matrix $\mathbf{A}^+$ formulated as

$$\mathbf{A}^+ = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T. \tag{27}$$

Some OLPPRFBs have used this pseudo inverse [3]. In the perspective to yield the unique inverse, the method is suitable since the uniqueness is always guaranteed. However, there are two problems on the Type x.3 OLPPRFBs. One is the reduction of the design freedom: A rectangular matrix has a non-unique inverse which means there are multiple inverse matrices for the same rectangular matrix. It implies there may exist freedom to parameterize a left-inverse of the matrix. The pseudo inverse method eliminates its freedom. The other is the filter length condition: If there is an order-N $\mathbf{\Lambda}_{OH}(z)$, its pseudo inverse is not always order-N, which yields *different length* synthesis filters from the analysis bank. In this paper we consider OLPPRFBs with the same filter length in both banks.

To use the design freedom effectively and guarantee the order-N inverse, we modify the parameterization described in [2]. A $P \times M$ rectangular matrix $\mathbf{A}$ can be always divided into $M \times M$ invertible matrix $\mathbf{A}_0$ and $(P-M) \times M$ matrix $\mathbf{A}_1$ as follows:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_0 \\ \mathbf{A}_1 \end{bmatrix}. \tag{28}$$

Furthermore, there exists $\mathbf{A}^{-1}$ which is written into

$$\mathbf{A}^{-1} = \begin{bmatrix} \mathbf{A}_0^{-1} - \mathbf{A}_2\mathbf{A}_1\mathbf{A}_0^{-1} & \mathbf{A}_2 \end{bmatrix} \tag{29}$$

where $\mathbf{A}_2$ is an $M \times (P-M)$ arbitrary matrix. Consequently, the order-N synthesis building block condition can be imposed on $\mathbf{A}_2$. By using the structure in (25) and (26), $\hat{\mathbf{\Lambda}}_{OH}^{-1}(1)$ can be factorized into

$$\begin{aligned} &\hat{\mathbf{\Lambda}}_{OH}^{-1}(1) \\ &= \left(\begin{bmatrix} \mathbf{I}_{M/2} & \mathbf{0}_{M/2\times(P-M)/2} \end{bmatrix} + \mathbf{L}_{oh,1} \begin{bmatrix} -\mathbf{R}_{oh,1} & \mathbf{I}_{(P-M)/2} \end{bmatrix} \right) \\ &\times \operatorname{diag}(\mathbf{R}_{oh,0}^{-1}\mathbf{L}_{oh,0}^{-1}, \mathbf{I}_{(P-M)/2}) \end{aligned} \tag{30}$$

where

$$\mathbf{L}_{oh,1} = \begin{bmatrix} 0 & \cdots & \cdots \\ \vdots & \ddots & \\ 0 & \cdots & \cdots \\ l_{0,0} & 0 & \cdots \\ l_{1,0} & l_{1,1} & \ddots \\ \vdots & \vdots & \ddots \\ l_{(N-1)/2-1,0} & l_{(N-1)/2-1,1} & \cdots \end{bmatrix}. \tag{31}$$

IV. Design Example

In this section, we show a design example of an OLPPRFB with the proposed HOF building block based on the structure in Section III. It is optimized with one of the most popular cost functions to design filter banks, i.e., stopband attenuation, which is formulated as follows:

$$\begin{aligned} C_{ana.\ stpb.} &= \sum_{i=0}^{M-1} \int_{\omega \in i-th stopband} |H_i(e^{j\omega})|^2 d\omega \\ C_{syn.\ stpb.} &= \sum_{i=0}^{M-1} \int_{\omega \in i-th stopband} |F_i(e^{j\omega})|^2 d\omega \end{aligned} \tag{32}$$

where $C_{ana.\ stpb.}$ and $C_{syn.\ stpb.}$ are the stopband attenuations in the analysis and synthesis banks, respectively. In this paper, we set the entire cost function as the sum of these attenuations in both banks.

We designed a proposed OLPPRFB under the properties that $P = 8$, $M = 6$, $K_0 = K_1 = 1$, and $N = 3$ ($L = 36$). The frequency response of the analysis bank is shown as solid lines in Fig. 3. Also the response of the pre- and postprocessing system [6] with $P = 8$, $M = 6$, $K_0 = 3$, and $K_1 = 2$ ($L = 36$) is depicted as dashed lines in the figure for the comparison of filter characteristices. They show comparable results to each other, whereas the number of design parameters in the proposed system is 79, which is much fewer than 89 of the pre- and postprocessing system.

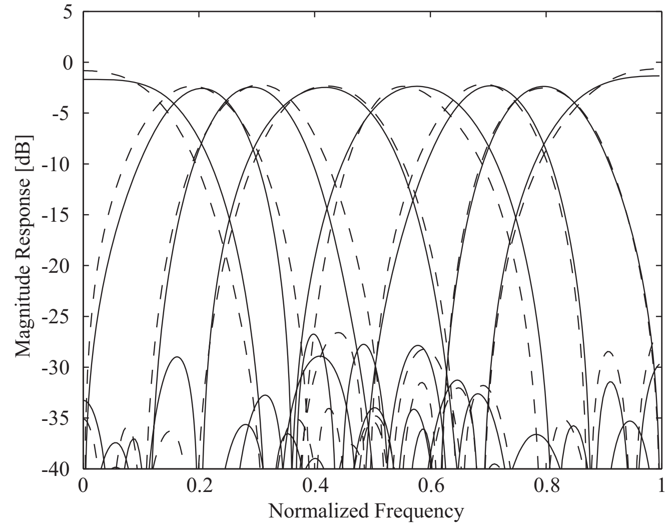

Fig. 3. Comparison between Design examples for the analysis banks of the Type 1.3 OLPPRFB with $K_0 = K_1 = 1$ and $N = 3$ (solid line) and the pre- and postprocessing system [6] with $K_1 = 2$ (dashed line), both have $P = 8$, $M = 6$ and $L = 36$.

V. Conclusions

In this paper, we presented a new lattice component of OLPPRFBs called HOF building blocks. OLPPRFBs with HOF building blocks have some advantages: 1) they have smaller number of design parameters and 2) building blocks than the traditional OLPPRFBs. Additionally, parameterizations of rectangular matrices for the HOF structure are provided. These indicate that the proposed OLPPRFBs can be an alternative structure of the traditional ones without discarding their merits. The proposed building block is also regarded as an extension of the critically-sampled HOF structure. Compared to the traditional successful approach to factorize the polyphase matrices of FBs into order-1 building blocks, factorizing FBs into order-N will become an attractive alternative in practical signal processing due to small implementation costs.

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