# A Lattice Structure of Biorthogonal Linear-Phase Filter Banks With Higher Order Feasible Building Blocks

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***Abstract*—This paper proposes a lattice structure of biorthogonal linear-phase filter banks (BOLPFBs) using new building blocks which can obtain long filters with fewer number of building blocks than conventional ones. The structure is derived from a generalization of the building blocks of first-order LPFBs. Furthermore, the proposed building blocks are applicable for both even and odd number of channels. The resulting FBs have good performance in stopband attenuation and low implementation costs.**

***Index Terms*—Biorthogonal (BO) filter banks (FBs), first-order (FO) linear-phase FBs, higher order feasible (HOF) building blocks, lapped transforms.**

## I. Introduction

FILTER BANKS (FBs) and their applications in the wide area of signal processing have been studied for a few decades [1]–[3]. There are many properties depending on the requirements. In practical applications, linear-phase (LP) property is highly desirable since the symmetric extension can be used at signal boundaries. Moreover, perfect reconstruction (PR) is one of the most important properties especially in the field of signal compression. In this paper, we propose a new structure of $M$-channel LPPRFBs.

One of the most efficient approaches to implement FBs is the lattice structure [1], [4]–[6]. It is based on a factorization of polyphase matrices of FBs. If high-order FBs are desired, they can be realized by cascading lower ordered building blocks. Usually order-1 (the highest order of $z^{-1}$ is one) building blocks are adopted as the lowest order [4]–[6]. The lattice structure of cascaded order-1 building blocks is effective in the viewpoint of achieving any-order FBs. However, the high-order FB requires many order-1 building blocks which increase the implementation cost. Furthermore, there exists some $M$-channel LPPRFBs which can *not* be factorized into the product of order-1 building blocks when the filter length is longer than $2M$ [7]. In other words, there may be a lattice factorization of order-$N$ $(N \geq 2)$ building blocks for LPPRFBs.

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Therefore, finding the efficient structure of order-$N$ building blocks is an interesting research problem. Generally it is very complicated since a general order-$N$ building block is composed of a matrix polynomial of delay elements $z^{-i}$ ($i$: some integer). The problem is to guarantee the LP property and calculate its inverse with a reasonable cost for the synthesis bank.

First-order (FO) LPFBs and their simplified structure have been studied [8]–[10]. They are generalized versions of $M$-channel biorthogonal (BO) LPFBs with all the filter length being $2M$ (denoted as $M \times 2M$, hereafter) [4], where the synthesis filter lengths can be longer than those of analysis filters. In other words, each of them has an order-1 analysis building block and an order-$N$ $(N \geq 1)$ synthesis building block. They have shown better results than the traditional BOLPFBs for image coding [10], with restriction that the longest analysis filter length is limited to $2M$. However, if we desire longer analysis filters for better frequency characteristic, FOLPFBs can not be adopted due to the restriction of the analysis filter length. In spite of the disadvantage, they have an attractive feature and help to derive an efficient order-$N$ structure.

This paper introduces a new building block structure called higher order feasible (HOF) building block which permits higher ordered BOLPFBs with fewer building blocks than the cascaded order-1 structure. Furthermore, building blocks of BOLPFBs and FOLPFBs belong to subclasses of HOF building blocks. The proposed FBs can yield long filters with reasonable implementation costs.

This paper is organized as follows: Section II gives brief reviews of BOLPFBs and FOLPFBs. The structure of a restricted HOF building block called third-order building block is represented in Section III. The complete HOF structure is shown in Section IV, which includes both even and odd-channel solutions. Section V presents design examples of BOLPFBs by using the proposed HOF building blocks and the comparison with the traditional BOLPFB in image coding application. Section VI concludes the paper.

### A. Notations

The identity and reversal matrices are $\mathbf{I}$ and $\mathbf{J}$, respectively. Also, $\mathrm{diag}(\cdot)$ denotes a block diagonal matrix. For simplicity, we omit matrix sizes when they are obvious.

## II. Review

### A. BOLPFBs

Consider an $M \times KM$ BOLPFB [4]. A typical structure of a FB and its polyphase representation are shown in Fig. 1. Using

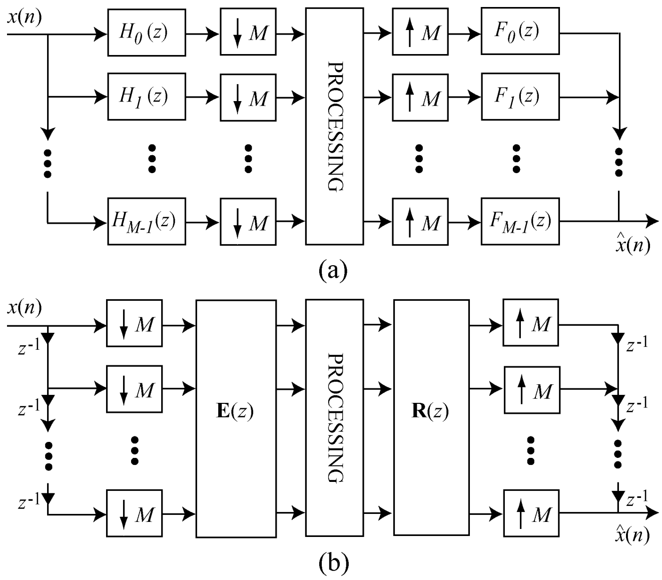


Fig. 1. $M$-channel maximally decimated FB. (a) Conventional representation. (b) Polyphase representation.

the lattice structure, the analysis polyphase matrix $\mathbf{E}(z)$ can be represented as

$$\mathbf{E}(z) = \mathbf{G}_{K-1}(z)\mathbf{G}_{K-2}(z)\ldots\mathbf{G}_1(z)\mathbf{E}_0. \quad (1)$$

If PR is achieved, the causal synthesis polyphase matrix $\mathbf{R}(z)$ is given as

$$\mathbf{R}(z) = z^{-(K-1)}\mathbf{E}_0^{-1}\mathbf{G}_1^{-1}(z)\mathbf{G}_2^{-1}(z)\ldots\mathbf{G}_{K-1}^{-1}(z). \quad (2)$$

When $M$ is even, each matrix in (1) is represented as follows:

$$\mathbf{G}_i(z) = \begin{bmatrix}\mathbf{U}_i & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_i\end{bmatrix}\mathbf{W}_e\mathbf{\Lambda}(z)\mathbf{W}_e \quad (3)$$

$$\mathbf{E}_0 = \begin{bmatrix}\mathbf{U}_0 & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_0\end{bmatrix}\mathbf{W}_e\begin{bmatrix}\mathbf{I}_{M/2} & \mathbf{0}_{M/2}\\ \mathbf{0}_{M/2} & \mathbf{J}_{M/2}\end{bmatrix} \quad (4)$$

where

$$\mathbf{W}_e = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{I}_{M/2} & \mathbf{I}_{M/2}\\ \mathbf{I}_{M/2} & -\mathbf{I}_{M/2}\end{bmatrix}, \quad \mathbf{\Lambda}(z) = \begin{bmatrix}z^{-1}\mathbf{I}_{M/2} & \mathbf{0}_{M/2}\\ \mathbf{0}_{M/2} & \mathbf{I}_{M/2}\end{bmatrix}.$$

If the $M/2 \times M/2$ matrices $\mathbf{U}_i$ and $\mathbf{V}_i$ are nonsingular, the FB is a BOLPFB. Furthermore, $\mathbf{V}_i$ for $i > 0$ can be set to $\mathbf{V}_i \equiv \mathbf{I}$ for simplicity without losing completeness [5]. The simplified lattice structure is shown in Fig. 2.

Furthermore, the simplified odd-channel BOLPFB is factorized as follows [5]:

$$\mathbf{E}(z) = \mathbf{G}_{K-2}(z)\mathbf{G}_{K-4}(z)\ldots\mathbf{G}_3(z)\mathbf{G}_1(z)\mathbf{E}_0 \quad (5)$$

$$\mathbf{G}_i(z) = \begin{bmatrix}\mathbf{U}_i & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{(M-1)/2}\end{bmatrix}\mathbf{W}_o\begin{bmatrix}z^{-1}\mathbf{I}_{(M-1)/2} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{(M+1)/2}\end{bmatrix} \times \mathbf{W}_o\begin{bmatrix}\mathbf{Q}_i & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{(M+1)/2}\end{bmatrix}\mathbf{W}_o \times \begin{bmatrix}z^{-1}\mathbf{I}_{(M+1)/2} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{(M-1)/2}\end{bmatrix}\mathbf{W}_o \quad (6)$$

$$\mathbf{E}_0 = \begin{bmatrix}\mathbf{U}_0 & \mathbf{0}\\ \mathbf{0} & \mathbf{Q}_0\end{bmatrix}\mathbf{W}_o\begin{bmatrix}\mathbf{I}_{(M+1)/2} & \mathbf{0}\\ \mathbf{0} & \mathbf{J}_{(M-1)/2}\end{bmatrix} \quad (7)$$

where the $\mathbf{U}_i$ and $\mathbf{Q}_i$ are $(M+1)/2 \times (M+1)/2$ and $(M-1)/2 \times (M-1)/2$ nonsingular matrices, respectively, and

$$\mathbf{W}_o = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{I}_{(M-1)/2} & \mathbf{0} & \mathbf{I}_{(M-1)/2}\\ \mathbf{0} & \sqrt{2} & \mathbf{0}\\ \mathbf{I}_{(M-1)/2} & \mathbf{0} & -\mathbf{I}_{(M-1)/2}\end{bmatrix}. \quad (8)$$

### B. FOLPFBs

In [9], the eigenstructure based characterization of $M$-channel BOLPFBs whose analysis lengths are $2M$ (they are called *FO*) and their synthesis filter lengths are equal to or longer than $2M$ was presented. Its lattice structure of the analysis bank is

$$\mathbf{E}(z) = \begin{bmatrix}\mathbf{U}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_1\end{bmatrix}\bar{\mathbf{W}}_e\begin{bmatrix}\mathbf{I}_{M/2}z^{-1} - \mathcal{J} & \mathbf{0}_{M/2}\\ \mathbf{0}_{M/2} & \mathcal{J}z^{-1} - \mathbf{I}_{M/2}\end{bmatrix} \times\mathbf{W}_e\begin{bmatrix}\mathbf{U}_0 & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_0\end{bmatrix}\mathbf{W}_e\begin{bmatrix}\mathbf{I}_{M/2} & \mathbf{0}\\ \mathbf{0} & \mathbf{J}_{M/2}\end{bmatrix} \quad (9)$$

where $\bar{\mathbf{W}}_e = (1/\sqrt{2})\begin{bmatrix}\mathbf{I}_{M/2} & -\mathbf{I}_{M/2}\\ \mathbf{I}_{M/2} & \mathbf{I}_{M/2}\end{bmatrix}$, each $\mathbf{U}_i$ and $\mathbf{V}_i$ is an $M/2 \times M/2$ nonsingular matrix, and $\mathcal{J}$ is an $M/2 \times M/2$ block diagonal with Jordan blocks of size $b_i$ ($i = 0, \ldots, n$, $b_i$ is nonincreasing positive integer and $\sum_{i=0}^{n} b_i = M/2$) with zero eigenvalue. For example, if $M = 6$ and $\{b_i\} = \{2, 1\}$, then $\mathcal{J} = \left[\begin{array}{cc|c}0 & 1 & 0\\ 0 & 0 & 0\\ \hline 0 & 0 & 0\end{array}\right]$. Moreover, $\mathbf{R}(z)$ is obtained as follows:

$$\mathbf{R}(z) = z^{-b_0}\begin{bmatrix}\mathbf{I}_{M/2} & \mathbf{0}\\ \mathbf{0} & \mathbf{J}_{M/2}\end{bmatrix}\mathbf{W}_e\begin{bmatrix}\mathbf{U}_0^{-1} & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_0^{-1}\end{bmatrix}\mathbf{W}_e \times\begin{bmatrix}\mathbf{I}_{M/2}z + \sum_{i=2}^{b_0}\mathcal{J}^{i-1}z^{i} & \mathbf{0}_{M/2}\\ \mathbf{0}_{M/2} & -\mathbf{I}_{M/2} - \sum_{i=1}^{b_0-1}\mathcal{J}^{i}z^{-i}\end{bmatrix} \times\bar{\mathbf{W}}_e^{T}\begin{bmatrix}\mathbf{U}_1^{-1} & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_1^{-1}\end{bmatrix}. \quad (10)$$

In this structure, some patterns of the synthesis filter length can be permitted. If $M = 6$, we can design an FOLPFB whose analysis filter length is $2 \times 6 = 12$ and synthesis length is 12 ($b_i = \{1, 1, 1\}$), 24 ($b_i = \{2, 1\}$) or 36 ($b_i = \{3\}$). The corresponding lattice structure is shown in Fig. 3. For further details on this class of FBs, please refer to articles [8], [9]. Obviously, when $b_i = \{1, \ldots, 1\}$, the resulting FB is a BOLPFB.

## III. Third-Order BOLPFBs as Generalization of FOLPFBs

In this section, to obtain the HOF building blocks, first we introduce a generalization of the building block of FOLPFBs. The generalized building block permits to have a few choices of filter lengths with one building block up to third order. The third-order building block is the simplest structure of the proposed HOF building blocks.

A block-based or time-domain structure of FBs is a good implementation to consider a signal processing framework [11], [12]. In this paper, the structure helps to realize and understand the proposed HOF building block. The block-based frameworks in the analysis sides of a BOLPFB and a FOLPFB are depicted in Fig. 4(a) and (b), respectively. For simplicity, both

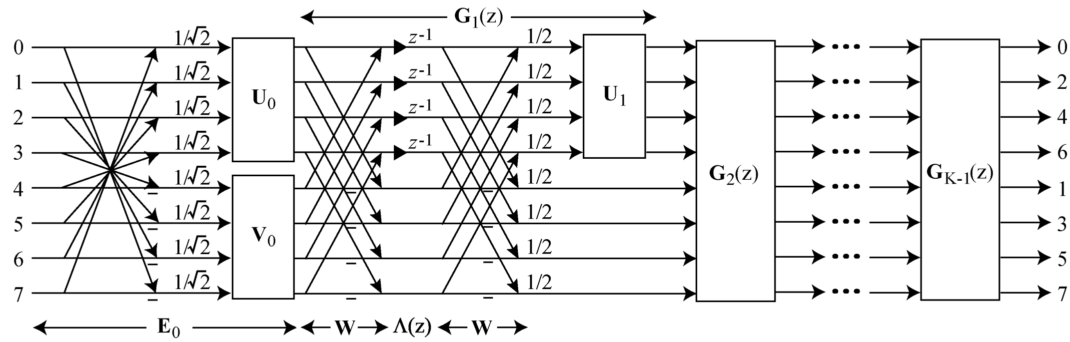


Fig. 2. Lattice structure of an even-channel analysis LPFB (shown for $M = 8$).

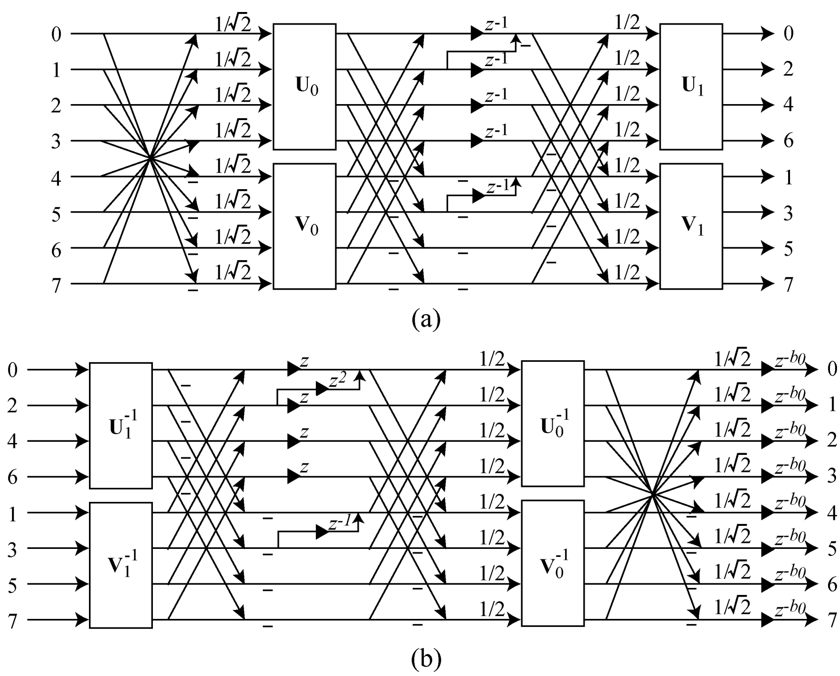


Fig. 3. Lattice structure of a FOLPFB ($b_i = \{2, 1, 1\}$, shown for $M = 8$). (a) Analysis bank. (b) Synthesis bank.

figures are shown for $M = 4$, $K = 2$ for the BOLPFB and $b_i = \{2\}$ for the FOLPFB. In these figures, $\mathbf{P}_0 = \mathbf{W}_e\mathbf{E}_0$, $\mathbf{P}_1 = \mathrm{diag}(\mathbf{U}_1, \mathbf{I})\mathbf{W}_e$ and $\mathbf{P}_2 = \mathrm{diag}(\mathbf{U}_1, \mathbf{V}_1)\bar{\mathbf{W}}_e$. It is easily understood that delay elements act as a shift operator at boundaries of each $M \times M$ block transform. Note that the difference between BOLPFBs and FOLPFBs is the existence of lifting operators between two block transforms. To generalize the structure of FOLPFBs, it is natural to focus on the lifting steps.

The typical lifting step for two signals consists of one prediction and one update operators [13]. With this consideration, the matrix with delay elements of FOLPFBs seems to be omitted either a prediction operator or an update one with its coefficients. Conversely, more general structure can be achieved by adding these operators and coefficients. Consequently, the lattice structure with the generalized building block is represented as follows:

$$\mathbf{E}(z) = \begin{bmatrix} \mathbf{U}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_1 \end{bmatrix} \bar{\mathbf{W}}_e \mathbf{\Lambda}_{TO}(z) \mathbf{W}_e \begin{bmatrix} \mathbf{U}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_0 \end{bmatrix} \mathbf{W}_e \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{J} \end{bmatrix} \tag{11}$$

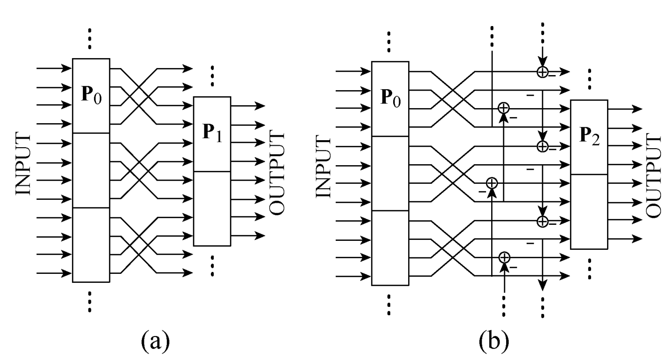


Fig. 4. Time-domain FB structure (shown for $M = 4$, analysis bank). (a) BOLPFB. (b) FOLPFB.

where

$$\mathbf{\Lambda}_{TO}(z) = \begin{bmatrix} z^{-2}\hat{\mathbf{\Lambda}}_{TO}(z) & \mathbf{0}_{M/2} \\ \mathbf{0}_{M/2} & -z^{-1}\hat{\mathbf{\Lambda}}_{TO}(z^{-1}) \end{bmatrix}$$

$$\hat{\mathbf{\Lambda}}_{TO}(z) = \begin{bmatrix} 1 & 0 & & \\ -p_0 z^{-1} & 1 & 0 & \\ 0 & -p_1 z^{-1} & 1 & \ddots \\ & \ddots & \ddots & \ddots \end{bmatrix} \times \begin{bmatrix} 1 & u_0 z & 0 & \\ 0 & 1 & u_1 z & \ddots \\ & 0 & 1 & \ddots \\ & & \ddots & \ddots \end{bmatrix}. \quad (12)$$

This building block has third order. Obviously if $p_i = 0$, the obtained FB is a FOLPFB (also if $p_i = u_i = 0$, the FB is a BOLPFB). The building block can yield wider range of filter lengths up to third order. However, the third-order building block is also a restricted version of the HOF ones. Furthermore, in this paper, we consider $M$-channel BOLPFBs with the same filter length in both analysis and synthesis banks. Unfortunately, the building block does not guarantee to yield the third-order inverse for the synthesis bank. In the next section, we derive the structure of the HOF building blocks based on the concept of the third-order BOLPFBs.

## IV. Formulation of HOF Building Blocks

In this section, we introduce the structures and properties of the proposed HOF building blocks of $M \times KM$ BOLPFBs. They can have up to $(M-1)$th-order with one building block. Furthermore, we show that any-order BOLPFBs can be designed by cascading the HOF building blocks. Also feasibility of odd-channel HOF building blocks is indicated.

### A. Structure of Even-Channel HOF Building Blocks

Before we formulate a HOF building block, consider the form in (12). The differences between block diagonal matrices in $\mathbf{\Lambda}_{TO}(z)$ are signs and powers of $z$. Furthermore, $\mathbf{\Lambda}_{TO}(z)$ maintains the lifting structure which implies that $\mathbf{\Lambda}_{TO}(z)$ belongs to a subclass of complete lifting matrices.

Generally an $M \times M$ nonsingular matrix $\mathbf{B}$ can always be factorized into $\mathbf{B} = \bar{\mathbf{P}}\bar{\mathbf{L}}\bar{\mathbf{D}}\bar{\mathbf{U}}$ called the LDU factorization, where all matrices have the size $M \times M$ and $\bar{\mathbf{P}}$ is a permutation matrix, $\bar{\mathbf{L}}$ is a lower triangular matrix, $\bar{\mathbf{D}}$ is a diagonal matrix, and $\bar{\mathbf{U}}$ is an upper triangular matrix [14]. By using the LDU matrix factorization, we assume the order-$N$ delay matrix as

$$\mathbf{\Lambda}^e_{\mathrm{HOF}}(z) = \begin{bmatrix} \hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(z) & \mathbf{0} \\ \mathbf{0} & -\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(z') \end{bmatrix} \quad (13)$$

where $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(1)$ is an $M/2 \times M/2$ invertible matrix using the LDU factorization and $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(z')$ has different powers of $z$ from $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(z)$. In this factorization, a general lifting structure can be realized.

We expect the factorization of a HOF building block to be similar to $\mathbf{\Lambda}_{TO}(z)$, thus we assume the structure as

$$\mathbf{G}^e_{\mathrm{HOF}}(z) = \begin{bmatrix} \mathbf{U} & \mathbf{0} \\ \mathbf{0} & \mathbf{V} \end{bmatrix} \bar{\mathbf{W}}_e \mathbf{\Lambda}^e_{\mathrm{HOF}}(z) \mathbf{W}_e \quad (14)$$

where $\mathbf{U}$ and $\mathbf{V}$ are $M/2 \times M/2$ nonsingular matrices, respectively. To derive the condition for $\mathbf{\Lambda}^e_{\mathrm{HOF}}(z)$ as a component of a BOLPFB, we consider the LP condition of the analysis polyphase matrix. If all filters of $\mathbf{E}(z)$ have lengths $KM$, the LP condition is represented as [1], [4]

$$\mathbf{E}(z) = \hat{\mathbf{D}} z^{K-1} \mathbf{E}(z^{-1}) \mathbf{J} \quad (15)$$

where $\hat{\mathbf{D}}$ is the diagonal matrix with entries $+1$ or $-1$, depending on whether the filter is symmetric $(+1)$ or antisymmetric $(-1)$. Under the assumption in (13), the condition can be simplified to a constraint of the order-$N$ delay matrix as

$$\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(z) = z^{-N+1} \hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(z'^{-1}). \quad (16)$$

The condition in (16) implies that the powers of $z$ in $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(z')$ are determined uniquely from those in $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(z)$.

Although the powers of $z$ in (16) could have various patterns, there are a few requirements for them to obtain good frequency response. First, all diagonal elements in $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(z)$ should have the same powers. Second, the difference between the diagonal power of $z$ in $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(z)$ and that in $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(z')$ should be $|1|$. Third, powers of $z$ should be changed gradually to yield consecutive filter coefficients. For example, there should not be $z^{-3}$ next to $z^{-1}$. We decide the powers and positions of $z$ by using these rules.

Another important rule is considered in this paper. That is, every element in $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(z)$ is a *monomial* of $z^{-i}$. It avoids cumbersome calculations to obtain the inverse of a HOF building block since the inverse can be obtained as the product of each element in $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(1)$ (a constant matrix) and powers of $z^{-i}$.

Next, we reduce the number of redundant parameters in $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(1)$. It is commonly known that positions of each matrix in the LDU decomposition can be changed without any disadvantage [15]. In this paper, we assume $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(1)$ can be decomposed into

$$\begin{aligned} \hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(1) &= \mathbf{P}_h \mathbf{D}_h \mathbf{L}_h \mathbf{R}_h \\ &= \mathbf{P}_h \begin{bmatrix} d_0 & 0 & & \\ 0 & d_1 & 0 & \\ & 0 & d_2 & \ddots \\ & & \ddots & \ddots \end{bmatrix} \begin{bmatrix} 1 & 0 & & \\ l_{10} & 1 & 0 & \\ l_{20} & l_{21} & 1 & \ddots \\ \vdots & \vdots & \ddots & \ddots \end{bmatrix} \\ &\times \begin{bmatrix} 1 & u_{01} & u_{02} & \cdots \\ 0 & 1 & u_{12} & \cdots \\ & 0 & 1 & \ddots \\ & & \ddots & \ddots \end{bmatrix}. \end{aligned} \quad (17)$$

Furthermore, $\mathbf{P}_h \mathbf{D}_h$ can be merged into the block diagonal diag$(\mathbf{U}, \mathbf{V})$ in (14) since

$$\bar{\mathbf{W}}_e \begin{bmatrix} \mathbf{P}_h \mathbf{D}_h & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_h \mathbf{D}_h \end{bmatrix} = \begin{bmatrix} \mathbf{P}_h \mathbf{D}_h & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_h \mathbf{D}_h \end{bmatrix} \bar{\mathbf{W}}_e \quad (18)$$

is always true. Consequently, $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(1) = \mathbf{L}_h \mathbf{R}_h$ for a HOF building block. Additionally, the structure of $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(1)$ is restricted to obtain the same filter length in both banks. If we need an order-$N$ HOF building block, $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(1)$ is limited to be the block diagonal matrix whose maximum (and at least one) block

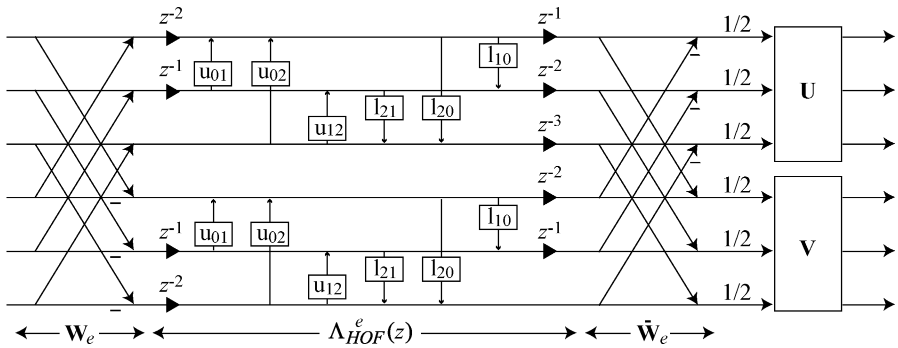


Fig. 5. Structure of even-channel analysis HOF building block (shown for $M = 6$ and fifth order).

size is $(N+1)/2$. Finally, the restriction implies that both $\mathbf{L}_h$ and $\mathbf{R}_h$ have to be the block diagonal matrices such as

$$\mathbf{L}_h = \begin{bmatrix} \mathbf{L}_{h,0} & \mathbf{0} & \\ \mathbf{0} & \mathbf{L}_{h,1} & \ddots \\ & \ddots & \ddots \end{bmatrix}, \quad \mathbf{R}_h = \begin{bmatrix} \mathbf{R}_{h,0} & \mathbf{0} & \\ \mathbf{0} & \mathbf{R}_{h,1} & \ddots \\ & \ddots & \ddots \end{bmatrix} \tag{19}$$

where $\mathbf{L}_{h,n}$ and $\mathbf{R}_{h,n}$ $(n = 0, 1, \ldots)$ are $\tilde{N} \times \tilde{N}$ $(\tilde{N} \leq (N+1)/2)$ lower and upper triangular matrices whose all diagonal elements are 1, respectively. Afterwards, we assume the size of $\mathbf{L}_{h,0}$ and $\mathbf{R}_{h,0}$ is $((N+1)/2) \times ((N+1)/2)$ for simplicity.

By combining (16) and the structure of $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(1)$, we obtain the complete structure of $\mathbf{\Lambda}^e_{\mathrm{HOF}}(z)$ as follows:

$$\mathbf{\Lambda}^e_{\mathrm{HOF}}(z) = \begin{bmatrix} \hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(z_{i,j}) & \mathbf{0} \\ \mathbf{0} & -\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}\left(z'_{i,j}\right) \end{bmatrix}. \tag{20}$$

In (20), the power of the $(i,j)$th element in $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(z_{i,j})$ and $\hat{\mathbf{\Lambda}}_{\mathrm{HOF}}(z'_{i,j})$ is defined by $s_{i,j}$ and $s'_{i,j}$, respectively, where

$$s_{i,j} = \begin{cases} s_{i,i} = \frac{-(N+1)}{2} \\ s_{i,j+1} = s_{i,j} + 1\,, \\ s_{i+1,j} = s_{i,j} - 1 \end{cases} \quad s'_{i,j} = s_{j,i} + 1. \tag{21}$$

The building block structure is illustrated in Fig. 5. Moreover, the inverse of $\mathbf{G}^e_{\mathrm{HOF}}(z)$ is

$$\mathbf{G}^{e^{-1}}_{\mathrm{HOF}}(z) = \mathbf{W}_e \mathbf{\Lambda}^e_{\mathrm{IHOF}}(z) \bar{\mathbf{W}}_e^T \begin{bmatrix} \mathbf{U}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{V}^{-1} \end{bmatrix}. \tag{22}$$

$\mathbf{\Lambda}^e_{\mathrm{IHOF}}(z)$ is formulated as

$$\mathbf{\Lambda}^e_{\mathrm{IHOF}}(z) = \begin{bmatrix} \hat{\mathbf{\Lambda}}^e_{\mathrm{IHOF}}\left(z'_{j,i}\right) & \mathbf{0} \\ \mathbf{0} & -\hat{\mathbf{\Lambda}}^e_{\mathrm{IHOF}}(z_{j,i}) \end{bmatrix} \tag{23}$$

where $\hat{\mathbf{\Lambda}}^e_{\mathrm{IHOF}}(1) = \mathbf{R}_h^{-1}\mathbf{L}_h^{-1}$. If $\mathbf{R}_h = \mathbf{L}_h = \mathbf{I}$, the resulting HOF building block is the same as an order-1 building block. Furthermore, if $\mathbf{L}_h = \mathbf{I}$ and $(\mathbf{R}_h - \mathbf{I})$ is a block diagonal matrix with Jordan blocks, the building block is the same as a FOLPFBs' component. Consequently, these two traditional building blocks are special cases of the proposed HOF structure.

It can permit an odd-order (up to $(M-1)$th order) BOLPFB with one HOF building block. Furthermore, the building block does not need to calculate its inverse of the matrix polynomial for $\mathbf{G}^{e^{-1}}_{\mathrm{HOF}}(z)$ since the inverse can be easily obtained as a product of each element of $\hat{\mathbf{\Lambda}}^e_{\mathrm{IHOF}}(1)$ and $z'_{j,i}$ or $z_{j,i}$.

### B. Realization of Even-Order BOLPFBs With HOF Building Blocks

In the above subsection, we have formulated the HOF building block. However, only odd-order BOLPFBs are permitted by using a HOF building block. To realize even-order BOLPFBs is a challenging problem. Fortunately, the proposed building block $\mathbf{G}^e_{\mathrm{HOF}}(z)$ satisfies the order-$N$ LP building block condition $\mathbf{G}^e_{\mathrm{HOF}}(z) = z^{-N}\hat{\mathbf{D}}\mathbf{G}^e_{\mathrm{HOF}}(z^{-1})\hat{\mathbf{D}}$ [4]. It suggests that HOF building blocks can be cascaded without losing the LP property. The previous subsection also introduced that an order-1 building block belongs to a subclass of HOF building blocks. After all, even-order BOLPFBs can be realized by cascading order-1 and HOF building blocks.

### C. Structure of Odd-Channel HOF Building Blocks

Up to now, we have just considered the even-channel case. However, the odd-channel solution should not be omitted. The previous subsection shows that each HOF building block can be cascaded. It is useful to obtain the HOF building blocks for FB with odd $M$.

In [16], the permissible condition on the filter length and symmetry polarity for LPPRFBs has been presented. For an odd-channel LPPRFB with all the filter length being $KM$, the condition requires $K$ has to be odd. It means that only odd-channel even-order LPPRFBs can be designed. Hence, to realize an odd-channel HOF building block, even-order one is only considered. The structure is expected to be similar to the cascaded even-channel HOF building block and odd-channel solution for traditional BOLPFBs [4], [5]. Therefore, an odd-channel HOF structure can be factorized into

$$\mathbf{G}^o_{\mathrm{HOF}}(z) = \begin{bmatrix} \mathbf{U}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{(M-1)/2} \end{bmatrix} \mathbf{W}_o \begin{bmatrix} z^{-1}\mathbf{I}_{(M-1)/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{(M+1)/2} \end{bmatrix} \times \mathbf{W}_o \begin{bmatrix} \mathbf{Q}_{i,0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{i,1} \end{bmatrix} \mathbf{W}_o \mathbf{\Lambda}^o_{\mathrm{HOF}}(z) \mathbf{W}_o \tag{24}$$

where the $\mathbf{U}_i$, $\mathbf{Q}_{i,0}$ and $\mathbf{Q}_{i,1}$ are $(M+1)/2 \times (M+1)/2$, $(M-1)/2 \times (M-1)/2$ and $(M+1)/2 \times (M+1)/2$ nonsingular matrices, respectively. Moreover, $\mathbf{\Lambda}^o_{\text{HOF}}(z)$ is represented as follows:

$$\mathbf{\Lambda}^o_{\text{HOF}}(z) = \begin{bmatrix} \hat{\mathbf{\Lambda}}_{\text{HOF}}(z_{i,j}) & \mathbf{0}_{(M-1)/2\times 1} & \mathbf{0}_{(M-1)/2} \\ \mathbf{0}_{1\times(M-1)/2} & z^{\frac{-(N-1)}{2}} & \mathbf{0}_{1\times(M-1)/2} \\ \mathbf{0}_{(M-1)/2} & \mathbf{0}_{(M-1)/2\times 1} & -\hat{\mathbf{\Lambda}}_{\text{HOF}}\left(z'_{i,j}\right) \end{bmatrix} \tag{25}$$

where both $\hat{\mathbf{\Lambda}}_{\text{HOF}}(z_{i,j})$ and $\hat{\mathbf{\Lambda}}_{\text{HOF}}(z'_{i,j})$ have the size $(M-1)/2 \times (M-1)/2$. For this case, the starting block $\mathbf{E}_0$ has the same structure as a BOLPFB in (7).

### *D. Comparison of the Numbers of Required Building Blocks, Design Parameters and Delay Elements*

First, we compare the number of building blocks (except $\mathbf{E}_0$) with the traditional BOLPFBs. By using the HOF structure, the maximum number of the blocks can be two as long as the order of the FB is equal to or less than $M$. In other words, the structure could have various filter lengths within two HOF building blocks. Conversely, the number of building blocks for traditional BOLPFBs depends on their required filter lengths. In the even-channel case, it is easily calculated as $(K-1)$. Furthermore, an odd-channel BOLPFB has $(K-1)/2$ building blocks and each building block has two parameterized matrices $\mathbf{U}_i$ and $\mathbf{Q}_i$.

Next, we describe the number of design parameters for $\mathbf{G}^e_{\text{HOF}}(z)$ in (14). It could have some patterns due to the sizes of $\mathbf{L}_{h,j}$ and $\mathbf{R}_{h,j}$ $(j \geq 1)$. For example, when an $8 \times 32$ BOLPFB (third order) with a HOF building block is needed, the sizes of $\mathbf{L}_{h,i}$ (denoted as $\{\mathbf{L}_{h,i}\}$) could be $\{\mathbf{L}_{h,0}, \mathbf{L}_{h,1}\} = \{2,2\}$ or $\{\mathbf{L}_{h,0}, \mathbf{L}_{h,1}, \mathbf{L}_{h,2}\} = \{2,1,1\}$. However, the largest number (in this case, $\{\mathbf{L}_{h,0}, \mathbf{L}_{h,1}\} = \{2,2\}$) should be considered in this paper.

For simplicity, the number of design parameters in the third and fourth order is calculated. Those for higher order can be calculated with the same consideration. In one HOF building block, there are two nonsingular matrices $\mathbf{U}$ and $\mathbf{V}$. Each of them has $M^2/4$ design parameters. Then, parameters in $\mathbf{\Lambda}_{\text{HOF}}(z)$ in (20) are considered. From (19), the largest block size of the block diagonal matrix in $\hat{\mathbf{\Lambda}}^e_{\text{HOF}}(1)$ is $2 \times 2$ for third order, hence it has two design parameters. Furthermore, the number of the $2 \times 2$ blocks in $\hat{\mathbf{\Lambda}}^e_{\text{HOF}}(1)$ is $\lfloor M/4 \rfloor$ ($M/4$ for even $M/2$, $(M-2)/4$ for odd $M/2$) where $\lfloor \cdot \rfloor$ is a function that returns the largest integer equal to or less than $(\cdot)$. Consequently, the total number of design parameters is $M^2/2 + 2\lfloor M/4 \rfloor$ for the third order. In the fourth-order case, an additional order-1 building block is cascaded to the third-order one. Hence, $M^2/4$ more design parameters are required. The total for the fourth order is $3(M/2)^2 + 2\lfloor M/4 \rfloor$.

The comparison of design parameters from third to sixth order for the even-channel case is shown in Table I. It is obvious that the HOF structure always has fewer parameters than the traditional cascaded order-1 structure. Generally many design parameters yield better filter responses, however, the amount of the improvement and the eliminable number of parameters are trade-off issues. Particularly if large number of channels or long filters are required, the reduction of the number of design parameters is much recommended than slightly modification of filter performance.

TABLE I
COMPARISON OF THE NUMBER OF DESIGN PARAMETERS

| Order | Cascaded Order-1 | HOF Build. blocks |
|---|---|---|
| 3 | $\frac{3}{4}M^2$ | $\frac{M^2}{2} + 2\lfloor \frac{M}{4} \rfloor$ |
| 4 | $M^2$ | $3\left(\frac{M}{2}\right)^2 + 2\lfloor \frac{M}{4} \rfloor$ |
| 5 | $\frac{5}{4}M^2$ | $\frac{M^2}{2} + 2\left(3\lfloor \frac{M}{6} \rfloor + \lfloor \frac{(M \bmod 6)}{4} \rfloor\right)$ |
| 6 | $\frac{3}{2}M^2$ | $3\left(\frac{M}{2}\right)^2 + 2\left(3\lfloor \frac{M}{6} \rfloor + \lfloor \frac{(M \bmod 6)}{4} \rfloor\right)$ |

Finally, the number of delay elements are considered. Although the HOF building block has various patterns depending on the required filter length, we consider the most general case of $N = M-1$ which means $\{\mathbf{L}_{h,0}\} = \{\mathbf{R}_{h,0}\} = M/2$. In the case, the structure in (20) is factorized into

$$\begin{aligned} &\mathbf{\Lambda}^e_{\text{HOF}}(z) \\ &\quad = \text{diag}\left(z^{-1}, z^{-2}, \ldots, z^{-M/2}, z^{-(M/2-1)}, \ldots, z^{-1}, 1\right) \\ &\quad\quad \times \text{diag}(\mathbf{\Lambda}_{\text{HOF}}(1), \mathbf{\Lambda}_{\text{HOF}}(1)) \\ &\quad\quad \times \text{diag}\left(z^{-(M/2-1)}, \ldots, z^{-1}, 1, 1, z^{-1}, \ldots, z^{-(M/2-1)}\right). \end{aligned} \tag{26}$$

From the right side of the above equation, the total number of delay elements is $M^2/4$ (first term) $+(M(M-2))/4$ (third term) $= (M(M-1))/2$. In contrast, $(M-1)$ order-1 building blocks are needed for the traditional $M \times (N+1)M$ $(= M \times M^2)$ BOLPFB. Each of the order-1 blocks has $M/2$ delays, thus the total is $M/2 \times (M-1) = (M(M-1))/2$. Obviously it is the same as that of the HOF structure. In the order-$N < (M-1)$ cases, the same relationships can be obtained easily. Consequently, the number of the delay elements in a HOF building block is equal to that in the cascaded order-1 ones.

## V. DESIGN EXAMPLES AND IMAGE CODING APPLICATION

### *A. Design*

In this subsection, we present three different designs of BOLPFBs with HOF building blocks. All FBs are optimized to have good stopband attenuation which is formulated as [1]

$$C_{\text{ana.stpb.}} = \sum_{i=0}^{M-1} \int_{\omega \in i\text{thstopband}} \left|H_i(e^{j\omega})\right|^2 d\omega \tag{27}$$

$$C_{\text{syn.stpb.}} = \sum_{i=0}^{M-1} \int_{\omega \in i\text{thstopband}} \left|F_i(e^{j\omega})\right|^2 d\omega. \tag{28}$$

The design examples are listed as follows.

Design Example 1: 8-Channel 32-Tap (Third Order)

This is an example of the structure in Section IV-A. The FB has one $\mathbf{G}^e_{\text{HOF}}(z)$. The normalized frequency responses are shown as solid lines in Fig. 6. To compare with the conventional method, the frequency responses of the $8 \times 32$ BOLPFB with order-1 building blocks [4] are also depicted as dashed lines in the figure. One can see that the proposed BOLPFB shows worse attenuations in the frequency bands

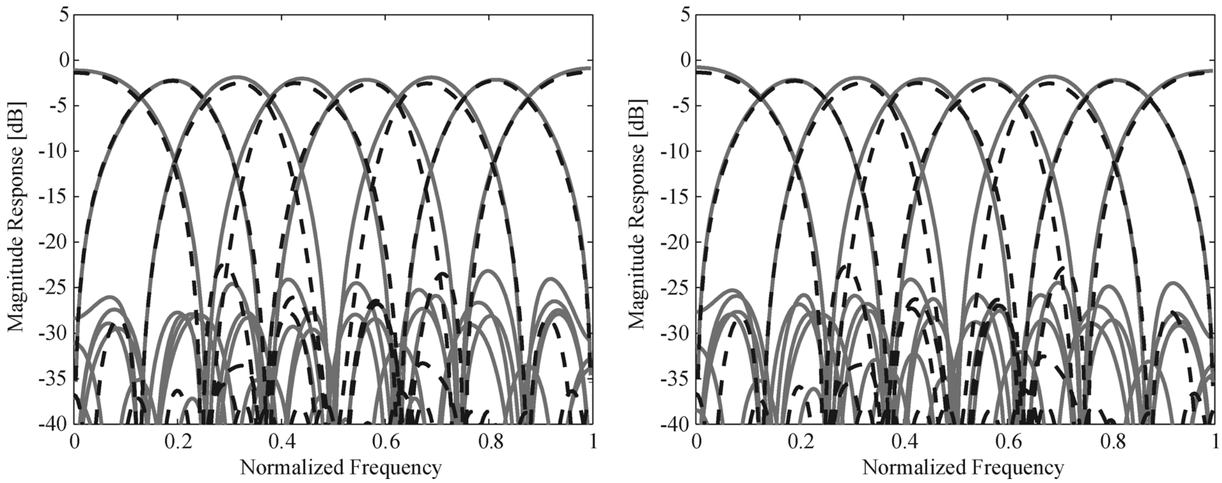


Fig. 6. Normalized frequency responses of the proposed 8 × 32 third-order BOLPFB (solid lines) and the traditional 8 × 32 BOLPFB with the cascaded order-1 structure (dashed lines). (Left) analysis bank. (Right) synthesis bank.

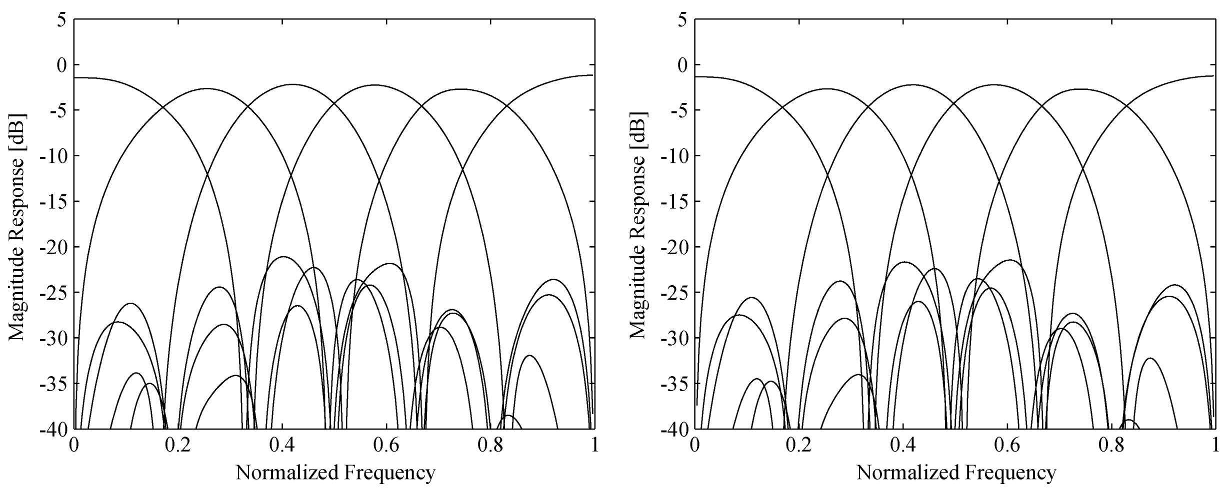


Fig. 7. Normalized frequency responses of the proposed 6 × 36 fifth-order BOLPFB. (Left) analysis bank. (Right) synthesis bank.

around $\omega = 0.2\pi$ and $0.8\pi$, whereas better in some frequencies (e.g., around $0.3\pi$ and $0.7\pi$) than that of the traditional one.

Since the HOF system has fewer building blocks and design parameters than the cascading order-1 structure, the frequency characteristics provide a good trade-off. Furthermore, in objective performance, the worst stopband attenuations in the analysis banks of the HOF structure and the order-1 structure are $-23.20$ (dB) and $-22.47$ (dB), respectively.

Design Example 2: 6-Channel 36-Tap (Fifth Order)

It has one $(M-1)$th-order $\mathbf{G}^{e}_{\mathrm{HOF}}(z)$ which is also described in Section IV-A. Fig. 7 depicts its frequency responses.

Design Example 3: 5-Channel 25-Tap (Fourth Order)

This example shows the odd-channel solution by using the HOF building block. The frequency responses are shown in Fig. 8. It has one order-4 $\mathbf{G}^{o}_{\mathrm{HOF}}(z)$.

It is observed that all examples have good stopband attenuations in both banks. As previously mentioned, BOLPFBs with the HOF building blocks always have fewer building blocks and design parameters than the traditional ones. It may lead to reduced implementation costs for not only design parameters but also hardware architecture. These advantages are the reasons for choosing the HOF building blocks.

### B. *Image Coding Application*

To validate our proposed method, image coding using FBs is suitable for fair comparison. Here we present the comparison in image coding with the traditional BOLPFB [4].

For image coding application, additional cost functions should be considered. In this paper, two additional functions, which are the coding gain and the DC leakage, are adopted. The generalized coding gain [17] is formulated as follows:

$$\begin{aligned} C_{cdg} &= 10\log_{10}\prod_{k=0}^{M-1}(A_kB_k)^{1/M} \\ A_k &= \sum_{n=0}^{KM-1}\sum_{m=0}^{KM-1} h_k(n)h_k(m)\rho^{|m-n|} \\ B_k &= \prod_{n=0}^{KM-1} f_k^2(n) \end{aligned} \tag{29}$$

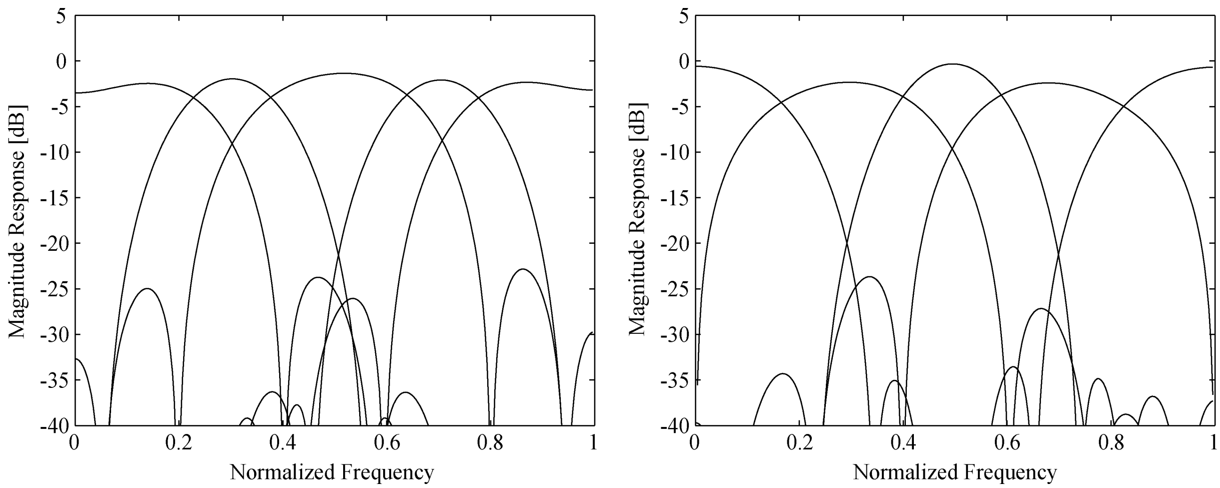


Fig. 8. Normalized frequency responses of the proposed 5 × 25 fourth-order BOLPFB. (Left) analysis bank. (Right) synthesis bank.

TABLE II
COMPARISON OF TRANSFORM PROPERTIES

| Transform Properties | 8 × 32 BOLPFB with order-1 building blocks | 8 × 32 BOLPFB with HOF building blocks |
|---|---|---|
| Coding Gain (dB) | 9.74 | 9.64 |
| Worst Stopband Attenuation (-dB) | 11.90 | 18.36 |
| Average Stopband Attenuation (-dB) | 12.39 | 20.56 |
| DC Attenuation (-dB) | 333.63 | 343.17 |

where $h_k(n)$ and $f_k(n)$ are filter coefficients of an analysis bank and a synthesis one, respectively, and $\rho$ is an autocorrelation factor of AR(1) process and is set to 0.95.

Furthermore, the DC leakage is an effective cost function for image coding since no DC leakage avoids checkerboard artifacts in compressed images [1]. The DC leakage is represented as

$$C_{DC} = \sum_{k=1}^{M-1} \sum_{n=0}^{KM-1} h_k(n). \tag{30}$$

We re-optimize two (conventional and proposed) 8 × 32 BOLPFBs shown in the previous subsection. They are optimized by the linear combination of the coding gain and the DC leakage. The filter coefficients optimized by the pure stopband attenuation have been set as the initial values of the FBs. The transform properties of the FBs are denoted in Table II. The HOF structure has 0.1 dB worse coding gain, however, has better stopband and DC attenuations than the cascaded order-1 structure. Since the proposed building block is *not* a simplified structure of that in the traditional BOLPFBs, i.e., it does not cover the same class as [4]–[6], the reduced design parameters would affect to the coding gain as mentioned previously. In contrast, the reduction hardly affects the performance in the other two cost functions. The comparison of frequency characteristics optimized for image coding is shown in Fig. 9.

The set partitioning in hierarchical trees (SPIHT) progressive image transmission algorithm [18] was used to encode the transformed images. The comparison of the image coding results is summarized in Table III. Two images *Lena* and *Barbara* representing images with smooth regions and textures respectively, were used in the comparison. Furthermore, the 8 × 16 BOLPFB with one order-1 building block, so-called the 8 × 16 GLBT [4], is also compared with the proposed BOLPFB since the order-1 system is often used for an image coding benchmark. The BOLPFB with the HOF building block presents better results than the traditional 8 × 32 BOLPFB. It also yields better results than the 8 × 16 BOLPFB for *Lena*

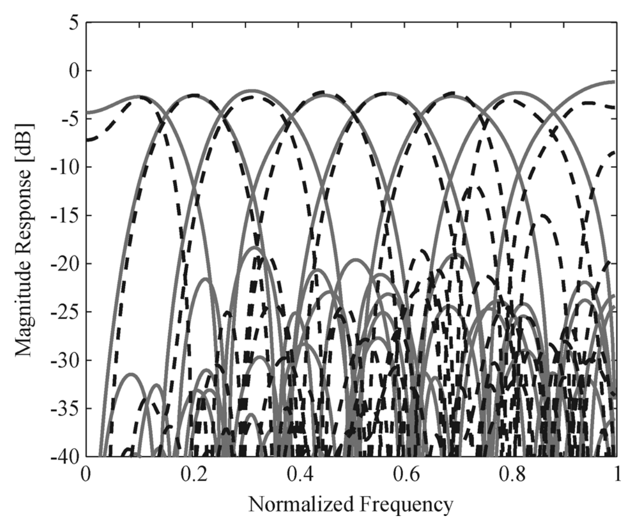


Fig. 9. Normalized frequency responses of the proposed 8 × 32 third-order BOLPFB (solid lines) and the traditional 8 × 32 BOLPFB with the cascaded order-1 structure (dashed lines). Both are the analysis banks and optimized for image coding application in Section V-B.

TABLE III
OBJECTIVE PERFORMANCE COMPARISON IN IMAGE CODING (PSNR IN DECIBLES)

| Lena | | | |
|---|---|---|---|
| Comp. ratio | $8 \times 16$ BOLPFB with order-1 build. Blocks | $8 \times 32$ BOLPFB with order-1 build. Blocks | $8 \times 32$ BOLPFB with HOF build. Blocks |
| 1:8 | 39.76 | 39.76 | 39.85 |
| 1:16 | 36.63 | 36.62 | 36.75 |
| 1:32 | 33.22 | 33.29 | 33.46 |
| **Barbara** | | | |
| Comp. ratio | $8 \times 16$ BOLPFB with order-1 build. Blocks | $8 \times 32$ BOLPFB with order-1 build. Blocks | $8 \times 32$ BOLPFB with HOF build. Blocks |
| 1:8 | 37.55 | 37.36 | 37.53 |
| 1:16 | 32.29 | 32.24 | 32.34 |
| 1:32 | 28.22 | 28.16 | 28.19 |

(especially 0.24-dB improvement on 32:1 compression), and comparable performance for *Barbara*.

The results between the $8 \times 32$ BOLPFB with HOF structure and the cascading order-1 one can be explained as follows. Roughly speaking, the large number of design parameters in FBs helps to yield good frequency characteristics, however, it makes difficult to get the optimal performance. In contrast, the HOF structure provides ease to obtain solution with the appropriate cost since it has fewer number of design parameters than the cascaded order-1 structure. Furthermore, the difference of the coding gain (0.1 dB) minorly affects image coding results. Especially in image coding, the important cost functions include the stopband attenuation and the DC leakage. The HOF structure shows better results in these two measures and consequently achieves better coding performance than the traditional $8 \times 32$ BOLPFB.

In contrast, the $8 \times 16$ BOLPFB sometimes shows slightly better PSNR than the $8 \times 32$ one with the HOF system. It can be explained by the well-known fact: Long filters provides less blocking artifacts in smooth regions (such as skin and background areas), whereas they often yields more ringing artifacts around edges of images [1]. It is clear that our long filters work well in *Lena* since it has a lot of smooth areas. Unfortunately, they also reduce the texture definition in *Barbara* image. Although there exists a trade-off between blocking and ringing artifacts, our proposed structure will be better for images which have a lot of smooth regions.

## VI. CONCLUSION

In this paper, we proposed a new structure of BOLPFBs by using HOF building blocks as a generalization of FOLPFBs. The proposed building blocks can be realized both even and odd-channel cases, thus it is regarded as an extension for the traditional structure of BOLPFBs. The building block is useful to design longer filters. Furthermore, the number of building blocks is fewer than that in the traditional cascaded order-1 structure which leads to reduced implementation costs in practical signal processing applications. In image coding application, the HOF structure shows better results than the cascaded order-1 structure. Our future work includes the derivation of more efficient structures.

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