A SIMPLIFIED LATTICE STRUCTURE OF FIRST-ORDER LINEAR-PHASE FILTER BANKS

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ABSTRACT

A simplified lattice structure for first-order linear-phase filter banks (FOLPFBs) is presented in this paper. A FOLPFB is a generalized version of biorthogonal linear-phase filter banks regarding their synthesis filter lengths. FOLPFBs' structure is more complicated and has more parameters than that in other FBs. We propose a method to reduce their redundant parameters without losing their properties. Moreover, regularity can be imposed which reduces the design freedom as well as improves the perceptual quality in image coding.

1. INTRODUCTION

There has been many researches in the field of filter banks (FBs) and their applications in multimedia signal processing. In signal compression, FBs are used to remove spatial correlation and the subband signals are quantized, coded and stored or transmitted. In this paper, we focus on FB design and its image coding application.

Makur *et al.* proposed another type of linear-phase (LP) FBs called first-order LPFBs (FOLPFBs) [1]. They are generalized versions of biorthogonal (BO) LPFBs [2] where the synthesis filter lengths can be longer than those of analysis filters. This property is useful for image compression since synthesis filters should be long and their coefficients should decay to zero smoothly at both ends to avoid blocking artifacts [3].

As the result of more general structure, they have more design freedom than that in traditional FBs. More freedom yields more flexibility in filter design which leads to better image coding performance. However, the number of free parameters is often not the actual design freedom since FBs often have redundant parameters. To design a FB, iteration of a nonlinear optimization is usually adopted. The optimization process depends on intial values of the filter coefficients. Thus the redundant parameters cause the optimization program to yield local minimal solutions. In this paper, we introduce a method to reduce the redundant parameters which solves this problem. Our proposed structure is general and covers other work presented in [4].

We impose a constraint, i.e., *regularity*, on our simplified FOLPFBs for image coding. It is an efficient restriction to accomplish both in improving perceptual visual quality of reconstructed images and in reducing design parameters. Regularity yields smooth basis functions of filters [3]. The first degree of regularity reduces checkerboarding artifacts and the second or higher degree of regularity controls the noises in a smoothness region of reconstructed images. The regularity conditions have already been imposed on BOLPFBs [5]. However, the condition for FOLPFBs has not been proposed yet and this issue is non-trivial. Here, we also propose regularity-constrained FOLPFBs.

Notations: The superscripts $\cdot^T$ and $\cdot^{-T}$ denote transposed and transposed inverse matrices, respectively. The identity matrix is $\mathbf{I}$, the reversal matrix is $\mathbf{J}$ and $\mathbf{1}_L$ is the $L\times 1$ column vector whose all values are 1. For simplicity, we omit vector or matrix sizes when they are obvious.

2. REVIEW

2.1 BOLPFBs

Consider an M-channel BOLPFB where all the filters have equal length KM [2]. By using the lattice structure, the analysis polyphase matrix $\mathbf{E}(z)$ can always be represented as

$$\mathbf{E}(z) = \mathbf{G}_{K-1}(z)\mathbf{G}_{K-2}(z)\ldots\mathbf{G}_1(z)\mathbf{E}_0. \tag{1}$$

If perfect reconstruction is achieved, the causal synthesis polyphase matrix $\mathbf{R}(z)$ is given as

$$\mathbf{R}(z) = z^{-(K-1)}\mathbf{E}_0^{-1}\mathbf{G}_1^{-1}(z)\mathbf{G}_2^{-1}(z)\ldots\mathbf{G}_{K-1}^{-1}(z). \tag{2}$$

When M is even, each matrix in (1) is represented as follows:

$$\mathbf{G}_i(z) = \mathbf{\Phi}_i\mathbf{W}\mathbf{\Lambda}(z)\mathbf{W}, \qquad \mathbf{E}_0 = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{U}_0 & \mathbf{U}_0\mathbf{J}\\ \mathbf{V}_0 & -\mathbf{V}_0\mathbf{J}\end{bmatrix} \tag{3}$$

where $\mathbf{\Phi}_i = \mathrm{diag}(\mathbf{U}_i, \mathbf{V}_i)$ and

$$\mathbf{W} = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{I}_{M/2} & \mathbf{I}_{M/2}\\ \mathbf{I}_{M/2} & -\mathbf{I}_{M/2}\end{bmatrix}, \quad \mathbf{\Lambda}(z) = \begin{bmatrix}\mathbf{I}_{M/2} & \mathbf{0}_{M/2}\\ \mathbf{0}_{M/2} & z^{-1}\mathbf{I}_{M/2}\end{bmatrix}.$$

If the $M/2\times M/2$ matrices $\mathbf{U}_i$ and $\mathbf{V}_i$ are nonsingular, the FB is a BOLPFB. Furthermore, $\mathbf{U}_i$ for $i>0$ can be set to $\mathbf{U}_i \equiv \mathbf{I}$ for simplicity without losing completeness [4].

2.2 FOLPFBs

In [1], the eigenstructure based characterization of M-channel BOLPFBs whose analysis filter lengths are $2M$ (they are called *first-order*) and synthesis ones are equal to or longer than $2M$ was presented. Its lattice structure of the analysis bank is

$$\begin{aligned}\mathbf{E}(z) =& \mathrm{diag}(\mathbf{A}_1,\mathbf{A}_2)\mathbf{W}'\begin{bmatrix}\mathbf{I}_{M/2}z^{-1} - \mathscr{J}_F & \mathbf{0}_{M/2}\\ \mathbf{0}_{M/2} & \mathscr{J}_F z^{-1} - \mathbf{I}_{M/2}\end{bmatrix}\\ &\times \mathbf{W}\mathrm{diag}(\mathbf{A}_3,\mathbf{A}_4)\mathbf{W}\mathrm{diag}(\mathbf{I}_{M/2},\mathbf{J}_{M/2})\end{aligned} \tag{4}$$

where $\mathbf{W}' = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{I}_{M/2} & -\mathbf{I}_{M/2}\\ \mathbf{I}_{M/2} & \mathbf{I}_{M/2}\end{bmatrix}$. Each $\mathbf{A}_i$ $(i=1,\ldots,4)$ is an $M/2\times M/2$ nonsingular matrix and $\mathscr{J}_F$ is an $M/2\times M/2$ block diagonal with Jordan blocks of size b_i $(i=0,\ldots,n,\ b_i$ is nonincreasing positive integer and $\sum_{i=0}^n b_i = M/2)$ with zero eigenvalue. For example, if $M=6$ and $\{b_i\}=\{2,1\}$, then $\mathscr{J}_F = \left[\begin{array}{cc|c}0&1&0\\0&0&0\\ \hline 0&0&0\end{array}\right]$.

Moreover $\mathbf{R}(z)$ is obtained as follows:

$$\begin{aligned}\mathbf{R}(z) =& z^{-b_0}\mathrm{diag}(\mathbf{I}_{M/2},\mathbf{J}_{M/2})\mathbf{W}\mathrm{diag}(\mathbf{A}_3^{-1},\mathbf{A}_4^{-1})\mathbf{W}\\ &\times\begin{bmatrix}\mathbf{I}_{M/2}z + \sum_{i=2}^{b_0}\mathscr{J}_F^{i-1}z^i & \mathbf{0}_{M/2}\\ \mathbf{0}_{M/2} & -\mathbf{I}_{M/2} - \sum_{i=1}^{b_0-1}\mathscr{J}_F^{i}z^{-i}\end{bmatrix}\\ &\times \mathbf{W}'^T\mathrm{diag}(\mathbf{A}_1^{-1},\mathbf{A}_2^{-1}).\end{aligned} \tag{5}$$

In this structure, some patterns of the synthesis filter length can be permitted. If $M = 6$, we can design a FOLPFB whose analysis filter length is $2\times 6 = 12$ and synthesis length is 12 ($b_i = \{1,1,1\}$), 24 ($b_i = \{2,1\}$) or 36 ($b_i = \{3\}$). For further information of this class of FBs, please refer to the article [1]. Obviously, when $b_i = \{1,\dots,1\}$, the obtained FB is a BOLPFB.

3. SIMPLIFIED FOLPFBS

In this section, we propose a simplified lattice structure of FOLPFBs which is our main contribution. Our structure guarantees to keep all properties of traditional FOLPFBs. Despite of various b_i's, the structure can eliminate redundant parameters.

3.1 Problem Statement

First, we consider the problem of eliminating redundancy. The lattice structure of FOLPFBs is different from those of other LPFBs because of $\mathscr{J}_F$ with FOLPFBs' delay elements. Thus we cannot straightforwardly apply the method used for other LPFBs.

Hence, we factorize $\mathbf{A}_1$ in the building block of $\mathbf{E}(z)$ into a product of two nonsingular matrices $\mathbf{A}_{10}$ and $\mathbf{A}_{11}$. If $\mathbf{A}_{10}$ has N parameters, obviously $\mathbf{A}_{11}$ has to have $\{(M/2)^2-N\}$ ones to keep all possible solutions. Conversely, if $\mathbf{A}_1$ has this structure, $\mathbf{E}(z)$ keeps its completeness. Furthermore, using a block diagonal matrix $\mathrm{diag}(\mathbf{A}_{11},\mathbf{A}_{11})$ yields the relationships

$$\begin{cases}\mathrm{diag}(\mathbf{A}_{11},\mathbf{A}_{11})\mathbf{W}'=\mathbf{W}'\mathrm{diag}(\mathbf{A}_{11},\mathbf{A}_{11})\\ \mathrm{diag}(\mathbf{A}_{11},\mathbf{A}_{11})\mathbf{W}=\mathbf{W}\mathrm{diag}(\mathbf{A}_{11},\mathbf{A}_{11}).\end{cases} \tag{6}$$

Consequently, the lattice structure of $\mathbf{E}(z)$ can be represented as

$$\begin{aligned}\mathbf{E}(z)=&\mathrm{diag}(\mathbf{A}_{10}\mathbf{A}_{11},\mathbf{A}_2)\mathrm{diag}(\mathbf{A}_{11}^{-1},\mathbf{A}_{11}^{-1})\mathrm{diag}(\mathbf{A}_{11},\mathbf{A}_{11})\\ &\times\mathbf{W}'\begin{bmatrix}\mathbf{I}_{M/2}z^{-1}-\mathscr{J}_F & \mathbf{0}_{M/2}\\ \mathbf{0}_{M/2} & \mathscr{J}_Fz^{-1}-\mathbf{I}_{M/2}\end{bmatrix}\mathbf{W}\\ &\times\mathrm{diag}(\mathbf{A}_3,\mathbf{A}_4)\mathbf{W}\mathrm{diag}(\mathbf{I},\mathbf{J})\\ =&\mathrm{diag}(\mathbf{A}_{10},\mathbf{A}_2\mathbf{A}_{11}^{-1})\mathbf{W}'\mathrm{diag}(\mathbf{A}_{11},\mathbf{A}_{11})\\ &\times\begin{bmatrix}\mathbf{I}_{M/2}z^{-1}-\mathscr{J}_F & \mathbf{0}_{M/2}\\ \mathbf{0}_{M/2} & \mathscr{J}_Fz^{-1}-\mathbf{I}_{M/2}\end{bmatrix}\mathbf{W}\\ &\times\mathrm{diag}(\mathbf{A}_3,\mathbf{A}_4)\mathbf{W}\mathrm{diag}(\mathbf{I},\mathbf{J}).\end{aligned} \tag{7}$$

From (7), if $\mathbf{A}_{11}$ satisfies the next equation, one can merge $\mathrm{diag}(\mathbf{A}_{11},\mathbf{A}_{11})$ into $\mathrm{diag}(\mathbf{A}_3,\mathbf{A}_4)$.

$$\begin{aligned}&\begin{bmatrix}\mathbf{A}_{11} & \\ & \mathbf{A}_{11}\end{bmatrix}\begin{bmatrix}\mathbf{I}_{M/2}z^{-1}-\mathscr{J}_F & \mathbf{0}_{M/2}\\ \mathbf{0}_{M/2} & \mathscr{J}_Fz^{-1}-\mathbf{I}_{M/2}\end{bmatrix}\\ =&\begin{bmatrix}\mathbf{I}_{M/2}z^{-1}-\mathscr{J}_F & \mathbf{0}_{M/2}\\ \mathbf{0}_{M/2} & \mathscr{J}_Fz^{-1}-\mathbf{I}_{M/2}\end{bmatrix}\begin{bmatrix}\mathbf{A}_{11} & \\ & \mathbf{A}_{11}\end{bmatrix}.\end{aligned} \tag{8}$$

The condition (8) can be simplified

$$\mathbf{A}_{11}\mathscr{J}_F=\mathscr{J}_F\mathbf{A}_{11}. \tag{9}$$

Our goal is to find $\mathbf{A}_{11}$ which satisfies the above equation.

3.2 Structure of $\mathbf{A}_{11}$

Next, the desired structure of $\mathbf{A}_{11}$ is presented. For convenience, let represent $\mathbf{A}_{11}$ as

$$\mathbf{A}_{11}=\begin{bmatrix}| & | & & |\\ \mathbf{p}_0 & \mathbf{p}_1 & \cdots & \mathbf{p}_{M/2-1}\\ | & | & & |\end{bmatrix}=\begin{bmatrix}— & \mathbf{q}_0 & —\\ — & \mathbf{q}_1 & —\\ & \vdots & \\ — & \mathbf{q}_{M/2-1} & —\end{bmatrix}. \tag{10}$$

Each side of (9) is rewritten by substituting (10) as follows:

$$\begin{aligned}\mathbf{A}_{11}\mathscr{J}_F&=\left[\begin{array}{cccc|cccc|c}| & | & & | & | & | & & | & \\ \mathbf{0} & \mathbf{p}_0 & \cdots & \mathbf{p}_{b_0-2} & \mathbf{0} & \mathbf{p}_{b_0} & \cdots & \mathbf{p}_{(b_0+b_1)-2} & \cdots\\ | & | & & | & | & | & & | & \end{array}\right]\\ \mathscr{J}_F\mathbf{A}_{11}&=\left[\begin{array}{cccc|cccc|c}| & & | & | & | & & | & | & \\ \mathbf{q}_0^T & \cdots & \mathbf{q}_{b_0-1}^T & \mathbf{0} & \mathbf{q}_{b_0+1}^T & \cdots & \mathbf{q}_{(b_0+b_1)-1}^T & \mathbf{0} & \cdots\\ | & & | & | & | & & | & | & \end{array}\right]^T.\end{aligned} \tag{11}$$

Comparing both equations in (11) yields the following theorem:

Theorem 1 *A nonsingular matrix $\mathbf{A}_{11}$ satisfies (9) if and only if it has the form*

$$\mathbf{A}_{11}=\left[\begin{array}{c|c|c}\mathbf{S}_{b_0} & \mathbf{T}_{b_0\times b_1} & \cdots\\ \hline \mathbf{T}_{b_1\times b_0} & \mathbf{S}_{b_1} & \ddots\\ \hline \vdots & \ddots & \ddots\end{array}\right], \tag{12}$$

where

$$\mathbf{S}_b=\begin{bmatrix}s_0 & s_1 & \cdots & s_{b-1}\\ 0 & s_0 & \ddots & \vdots\\ \vdots & \ddots & \ddots & s_1\\ 0 & \cdots & 0 & s_0\end{bmatrix},\mathbf{T}_{l\times m}=\begin{cases}\begin{bmatrix}\mathbf{S}_m\\ \mathbf{0}_{(l-m)\times m}\end{bmatrix} & l\geq m\\ \begin{bmatrix}\mathbf{0}_{l\times(m-l)} & \mathbf{S}_l\end{bmatrix} & m\geq l\end{cases}.$$

The proof is omitted since it can be proven by a direct calculation. A structure of $\mathbf{A}_{11}$ depends on a sequence b_i which also decides the number of eliminable parameters. For example, if $b_i=\{2,1,1\}$, $\mathbf{A}_{11}$ can be

$$\mathbf{A}_{11}=\left[\begin{array}{cc|c|c}s_0 & s_1 & s_2 & s_3\\ 0 & s_0 & 0 & 0\\ \hline 0 & s_4 & s_5 & s_6\\ \hline 0 & s_7 & s_8 & s_9\end{array}\right]. \tag{13}$$

Thus one can reduce 10 parameters from this FOLPFB. Conversely, $\mathbf{A}_{10}$ has just six ones. If $b_i=\{1,\dots,1\}$, $\mathbf{A}_{11}$ has all parameters of $\mathbf{A}_1$. It is the same as a simplified BOLPFB since $\mathbf{A}_{10}$ can be $\mathbf{I}$.

Consequently, the complete lattice structure of simplified FOLPFBs is represented as follows:

$$\begin{aligned}\mathbf{E}(z)=&\mathrm{diag}(\mathbf{A}_{10},\hat{\mathbf{A}}_2)\mathbf{W}'\begin{bmatrix}\mathbf{I}_{M/2}z^{-1}-\mathscr{J}_F & \mathbf{0}_{M/2}\\ \mathbf{0}_{M/2} & \mathscr{J}_Fz^{-1}-\mathbf{I}_{M/2}\end{bmatrix}\\ &\times\mathbf{W}\mathrm{diag}(\hat{\mathbf{A}}_3,\hat{\mathbf{A}}_4)\mathbf{W}\mathrm{diag}(\mathbf{I},\mathbf{J})\end{aligned} \tag{14}$$

where nonsingular matrices $\hat{\mathbf{A}}_2$, $\hat{\mathbf{A}}_3$ and $\hat{\mathbf{A}}_4$ are $\mathbf{A}_2\mathbf{A}_{11}^{-1}$, $\mathbf{A}_{11}\mathbf{A}_3$ and $\mathbf{A}_{11}\mathbf{A}_4$, respectively.

3.3 Parameterization

For various b_i's, it is difficult to find $\mathbf{A}_1$'s complete factorization into $\mathbf{A}_{10}\mathbf{A}_{11}$. In this paper, we factorize $\mathbf{A}_1$ by applying a lifting parameterization $\mathbf{A}_1=\mathbf{L}_1\mathbf{D}_1\mathbf{R}_1\mathbf{P}_1$ described in [5] recursively. This factorization is easy to find restricted parameters. For example, in the case of $b_i=\{2,1,1\}$, $\mathbf{A}_1$ can be factorized into

$$\begin{aligned}\mathbf{A}_1=&\overbrace{\begin{bmatrix}1 & 0 & 0 & 0\\ l_0 & 1 & 0 & 0\\ l_1 & 0 & 1 & 0\\ l_2 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \alpha_0 & r_0 & r_1\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}}^{\mathbf{A}_{10}}\\ &\times\overbrace{\begin{bmatrix}\mathbf{I}_2 & \\ & \mathbf{A}_{12}\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & l_3 & 1 & 0\\ 0 & l_4 & 0 & 1\end{bmatrix}\begin{bmatrix}\alpha_1 & \\ & \mathbf{I}_3\end{bmatrix}\begin{bmatrix}1 & r_2 & r_3 & r_4\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}}^{\mathbf{A}_{11}}\end{aligned} \tag{15}$$

where $\mathbf{A}_{12}$ is a 2×2 nonsingular matrix.

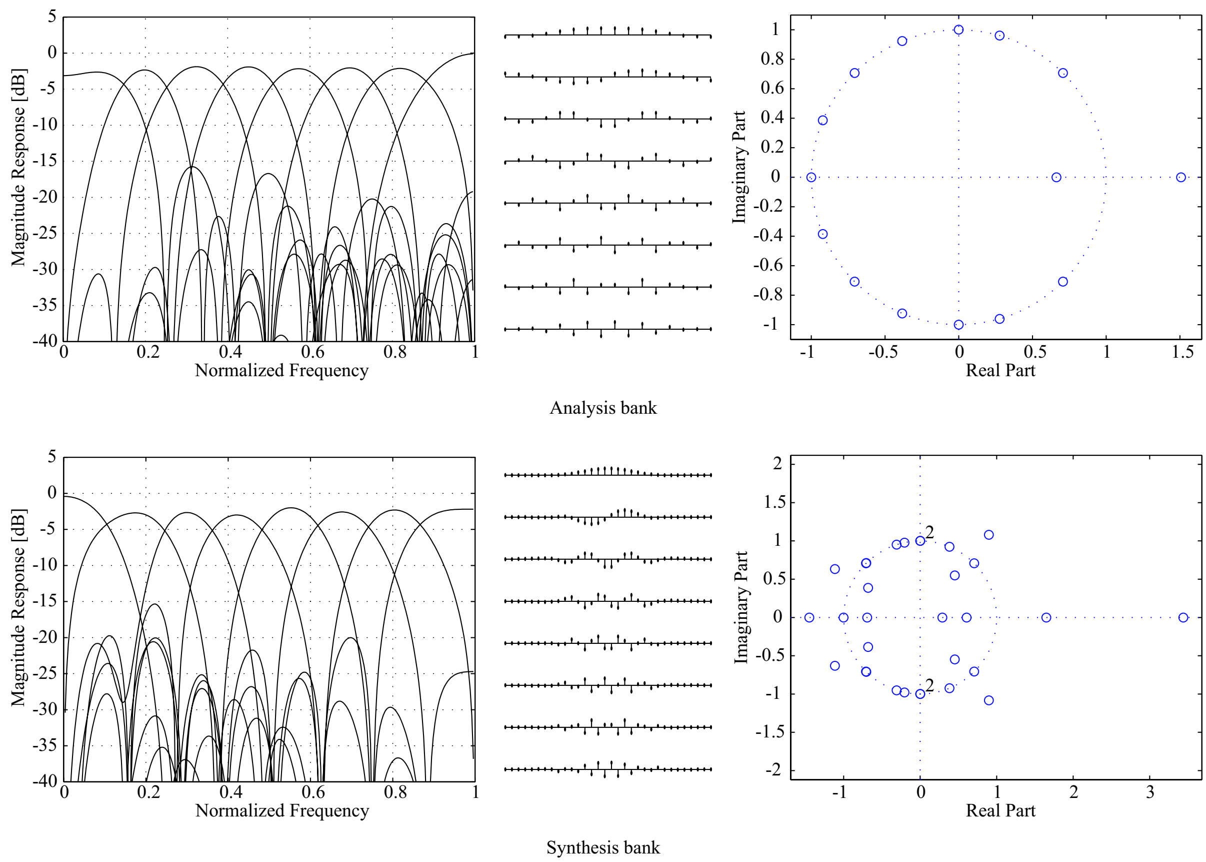

Figure 1: Design example I: (1, 1)-regular simplified FOLPFB with $b_i = \{2,1,1\}$. (Left) Frequency responses. (Middle) Impulse responses. (Right) Zero locations of the lowpass filters.

4. IMPOSING REGULARITY ON FOLPFBS

In this section, we also impose regularity on our simplified FOLPFBs. This paper introduces (1, 1) and (1, 2)-regular conditions for image coding. The regular FOLPFBs have slightly different structures from other FBs.

Initially, we denote the regularity condition for polyphase matrices of FBs. A filter bank is (K_a, K_s)-regular iff its polyphase matrices satisfy

$$\frac{d^n}{dz^n}\{\mathbf{E}(z^M)[1\ z^{-1}\ \ldots z^{1-M}]^T\}\Big|_{z=1} = c_n \mathbf{a}_M \quad (16)$$

$$\frac{d^m}{dz^m}\{\mathbf{R}^T(z^M)[z^{1-M}\ldots z^{-1}\ 1]^T\}\Big|_{z=1} = d_m \mathbf{a}_M \quad (17)$$

where $n = 0,\ldots,K_s-1$; $m = 0,\ldots,K_a-1$, $\mathbf{a}_M = [1\ 0\ldots\ 0]^T$ and c_n and d_m are some nonzero constants [5].

4.1 (1, 1)-regular Condition

To impose one-regular onto FOLPFBs, first we consider the (1, 0)-regular condition, and then the (1, 1)-regular one is derived due to a particular structure of $\mathbf{A}_{10}$. The (1, 0)-regular condition is calculated as follows from (17):

$$\mathbf{A}_{10}^{-T}(\mathbf{I}+\sum_{i=2}^{b_0}\mathscr{J}_F^{i-1})^T\hat{\mathbf{A}}_3^{-T}\mathbf{1}_{M/2} = d_0\mathbf{a}_{M/2}. \quad (18)$$

From Theorem 1, the first row of $\mathbf{A}_{10}^{-T}$ is always $[1\ 0\ \ldots 0]$, hence the effect of $\mathbf{A}_{10}^{-T}$ can be ignored in the above equation. Furthermore, we assume $\hat{\mathbf{A}}_3^{-T} = (\mathbf{I}+\sum_{i=2}^{b_0}\mathscr{J}_F^{i-1})^{-T}\mathbf{A}_3'^{-T}$ where

$$\mathbf{A}_3'^{-T} = \mathbf{R}_3\mathbf{D}_3\mathbf{L}_3\mathbf{P}_3 = \begin{bmatrix} 1 & r_{3,1} & \cdots & r_{3,M/2-1} \\ 0 & 1 & & \\ \vdots & & \ddots & \\ 0 & \cdots & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \mu_3 & \\ & \mathbf{B}_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & \cdots & 0 \\ l_{3,1} & 1 & & \vdots \\ \vdots & & \ddots & 0 \\ l_{3,M/2-1} & & & 1 \end{bmatrix} \mathbf{P}_3, \quad (19)$$

$\mu_3 \neq 0$, $\mathbf{B}_3$ is an $(M/2-1)\times(M/2-1)$ nonsingular matrix and $\mathbf{P}_3$ is a permutation matrix. From this factorization, we can impose the similar restriction to [5].

Next, we impose the (1, 1)-regular condition. The (0, 1)-regular one is denoted as

$$\mathbf{A}_{10}(\mathbf{I}-\mathscr{J}_F)\hat{\mathbf{A}}_3\mathbf{1}_{M/2} = c_0\mathbf{a}_{M/2}. \quad (20)$$

Therefore, the condition is rewritten as $\hat{\mathbf{A}}_{10}\mathbf{A}_3'\mathbf{1}_{M/2} = c_0\mathbf{a}_{M/2}$ where $\hat{\mathbf{A}}_{10} = \mathbf{A}_{10}(\mathbf{I}-\mathscr{J}_F)(\mathbf{I}+\sum_{i=2}^{b_0}\mathscr{J}_F^{i-1})$. The process to find the restriction is slightly different from [5], but the cumbersome detail is omitted in this paper because of avoiding any confusion and limitations of space. We only show the result for a (1, 1)-regular FOLPFB.

Condition 1 *If $\hat{\mathbf{A}}_3$ in (14) is factorized into (19), (1, 1)-regular simplified FOLPFBs can be designed with the following con-*

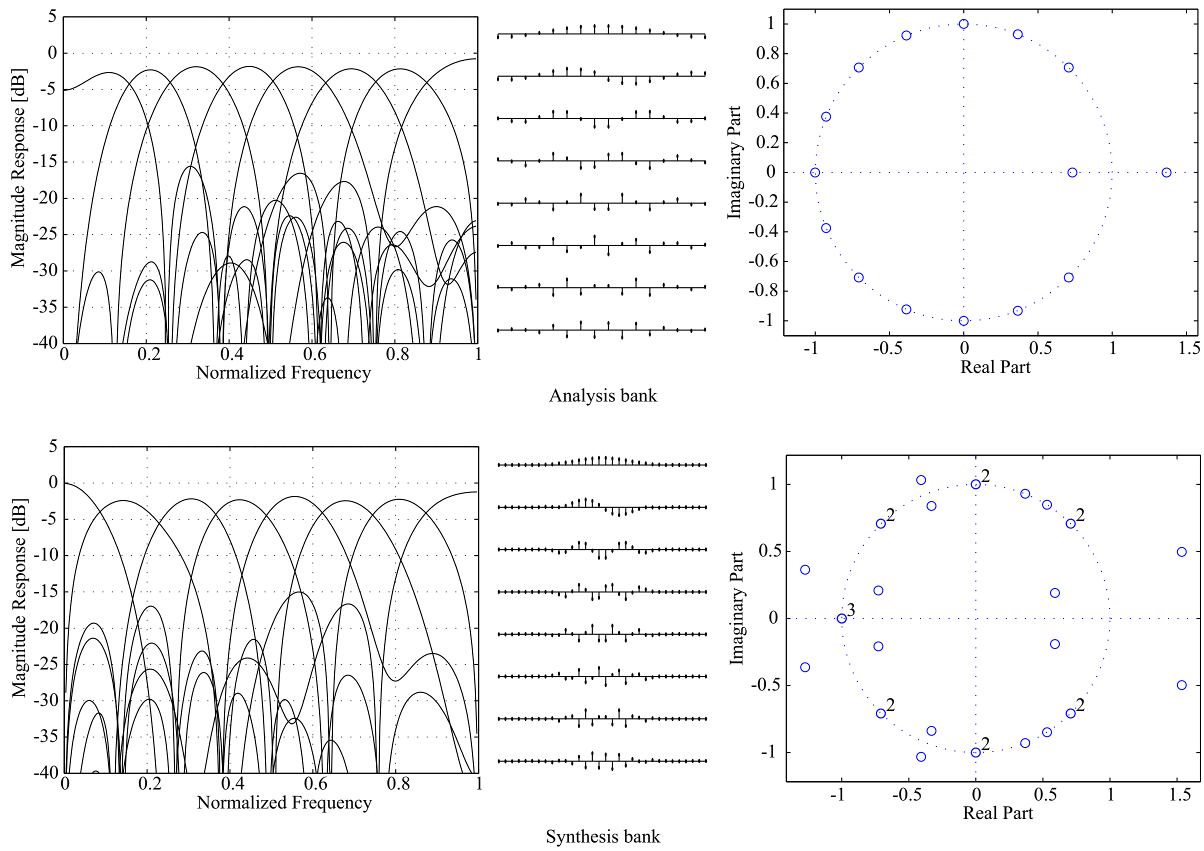

Figure 2: Design example II: (1, 2)-regular simplified FOLPFB with $b_i = \{2,1,1\}$. (Left) Frequency responses. (Middle) Impulse responses. (Right) Zero locations of the lowpass filters.

straints:

$$l_{3,i} = -1, \mu_3 = d_0, \text{and } \mathbf{B}_3^{-T}\mathbf{1} = \left(\mathbf{r}_3^T - \tilde{\mathbf{A}}_{10}^{-1}\mathbf{a}_{M/2-1}\right) c_0 \quad (21)$$

where $i = 1,\dots,M/2-1$, $\mathbf{r}_3 = [r_{3,1}\dots r_{3,M/2-1}]$ *and* $\tilde{\mathbf{A}}_{10}^{-1}$ *is* $\hat{\mathbf{A}}_{10}^{-1}$ *except its first row.*

4.2 (1, 2)-regular Condition

In this subsection we describe the condition for the (1, 2)-regular. From (16), we obtain the equation

$$(\mathbf{I} - \mathscr{J}_F)\hat{\mathbf{A}}_4\mathbf{b} - M(\mathbf{I} - \mathscr{J}_F)\hat{\mathbf{A}}_3\mathbf{1} = \mathbf{0} \quad (22)$$

where $\mathbf{b} = [M-1, M-3, \dots, 1]^T$. We assume $\hat{\mathbf{A}}_3$ has been already determined by the (1, 1)-regular condition. It means that $\hat{\mathbf{A}}_4$ has all constraints for the 2-regular in the synthesis bank. The constraints can be imposed onto $\hat{\mathbf{A}}_4$ with the lifting factorization $\hat{\mathbf{A}}_4 = \mathbf{L}_4\mathbf{D}_4\mathbf{R}_4\mathbf{P}_4$.

Condition 2 *If* $\hat{\mathbf{A}}_4$ *in (14) has the lifting structure* $\hat{\mathbf{A}}_4 = \mathbf{L}_4\mathbf{D}_4\mathbf{R}_4\mathbf{P}_4$, *(1, 2)-regular simplified FOLPFBs can be designed with the (1, 1)-regular condition and the following constraints:*

$$\mu_4 = t_0/b_r \text{ and } \mathbf{l}_4 = (\tilde{\mathbf{t}} - \mathbf{B}_4\tilde{\mathbf{b}})/t_0 \quad (23)$$

where $b_r = [1\ r_{4,1}\dots r_{4,M/2-1}]\mathbf{b}$, $\mathbf{t} = [t_0\ t_1\dots t_{M/2-1}]^T = M\hat{\mathbf{A}}_3\mathbf{1}$, $\tilde{\mathbf{t}} = [t_1\dots t_{M/2-1}]^T$, $\tilde{\mathbf{b}} = [M-3\dots 1]^T$, *and* $\mathbf{l}_4 = [l_{4,1}\dots l_{4,M/2-1}]^T$.

5. DESIGN EXAMPLES AND APPLICATION TO IMAGE CODING

In this paper, two simplified regular FOLPFBs are designed; one is the (1, 1)-regular and the other is the (1, 2)-regular. The cost function is the weighted linear combination of the coding gain and the stopband attenuation [3]. Both have eight-channel and $b_i = \{2,1,1\}$, thus they have 8×16 analysis and 8×32 synthesis FBs, respectively. Their frequency and impulse responses and zero locations of the lowpass filters are shown in Fig. 1 and 2, respectively.

The number of design parameters is one of the good measures to compare simplicity of FBs. The (1, 1)-regular FOLPFB and (1, 2)-regular one have 48 and 44 design parameters, respectively. These are fewer than the 8×16 BOLPFB's 48. Generally fewer parameters are desired since they lead to obtain the optimal solution faster. Furthermore, the coding gains of the proposed FBs are around 9.62 dB in spite of the restrictions. It is almost same as that of the BOLPFB. Hence the simplified regular FOLPFBs are useful both design and performance.

The proposed FOLPFBs are applied to image coding and compared to the performance of other FBs. We coded each transformed image by the embedded zerotree wavelet image codec presented in [6] for fair comparison. Coding results are summarized in Table 1 and reconstructed images are shown in Fig. 3. The (1, 1)-regular FOLPFB is observed to have superior coding results to the 8×16 regular BOLPFBs, as it has longer synthesis filters. The (1, 2)-regular FB has slightly worse results than the BOLPFBs in PSNRs, however, the reconstructed image is smooth (especially in the forehead areas) because of the 2-regular and long filters in its synthesis bank. Degradation of the image quality can be visible especially in smooth regions, thus the proposed (1, 2)-regular structure is effec-

Table 1: Comparison of image coding results (PSNR [dB]): LOT: [7], BOv11: 8×16 (1, 1)-regular BOLPFB, BOv12: 8×16 (1, 2)-regular BOLPFB, FOv11: Proposed (1, 1)-regular FOLPFB and FOv12: Proposed (1, 2)-regular FOLPFB.

Test images	Comp. ratio	Transforms				
		LOT	BOv11	BOv12	FOv11	FOv12
Barbara	1:32	**27.31**	27.06	26.95	27.29	26.84
	1:16	31.22	31.20	31.11	**31.43**	31.07
	1:8	35.67	35.81	35.69	**35.99**	35.64
Lena	1:32	29.89	**30.31**	29.46	**30.31**	29.36
	1:16	34.82	35.29	35.17	**35.42**	35.18
	1:8	38.41	38.71	38.61	**38.78**	38.63

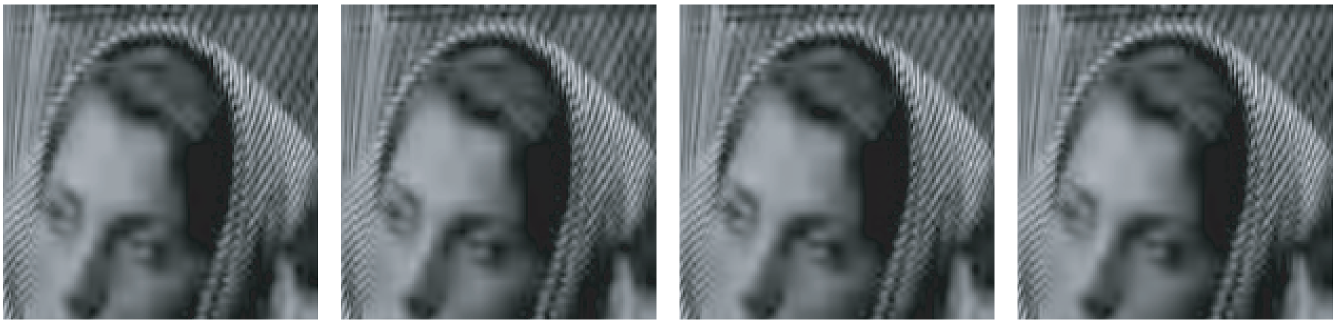

Figure 3: Enlarged images of *Barbara* (the compression ratio is 1:32). From left to right: 8×16 (1, 1)-regular BOLPFB [5]. 8×16 (1, 2)-regular BOLPFB [5]. (1, 1)-regular FOLPFB. (1, 2)-regular FOLPFB.

tive to improve perceptual visual qualities.

6. CONCLUSIONS

In this paper, we proposed a simplified lattice structure of FOLPFBs and their regularity constraints. Our simplified structure preserves all design freedom while eliminates redundant parameters. Furthermore in image coding application, our simplified regular FOLPFB yields better coding results than those of a BOLPFB in spite of having fewer free parameters. Our future work includes investigating better FOLPFB for image coding application.

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