DESIGN OF UNEQUAL-LENGTH LINEAR-PHASE FILTER BANKS WITHOUT REDUNDANCY

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ABSTRACT

In this paper, we present a new structure for linear-phase filter banks without redundancy, which have unequal-length filters at each subband. First, we extend the simplified lattice structure of the linear-phase filter bank to the unequal-length linear-phase paraunitary filter bank. In general, the unequal-length linear-phase *paraunitary* filter bank has equal-length filters at both the analysis and the synthesis bank because the synthesis bank is a transposed version of the analysis one, while the *biorthogonal* one does not. So, we discuss the conditions that the linear-phase biorthogonal filter bank has unequal-length filters at 1) the analysis bank, 2) the synthesis bank, and 3) both the analysis and the synthesis banks. Finally, several design and image coding examples are shown.

1. INTRODUCTION

Lapped transforms (LTs) play an important role in transform coding and have been extensively studied. From a filter bank perspective, the LTs are particular class of M-channel linear-phase perfect reconstruction filter banks, where the length of each filter L is KM (K is a positive integer) [3, 4].

The LT can considerably reduce the blocking effect due to the long basis function [9], which is a shortcoming of the DCT [1]; unfortunately, the long basis function is the main contributor to severe ringing around strong edges, where huge quantization errors are spread out to smoother neighbourhood regions. Therefore, for smooth image coding, it is desirable that the filter length should be long at low-frequency and short at high-frequency. One of the elegant solutions to this problem is to use unequal-length linear-phase filter banks (ULLPFBs) [5, 6]. They can achieve the huge reduction of the blocking and the ringing effect simultaneously. However, these structures are still redundant. In other words, such ULLPFBs have useless parameters. They may take long time to design the filter bank and the obtained solution may be local minimum.

In this paper, we present the design and implementation of an M-channel (M: even) unequal-length linear-phase filter bank without redundancy and approve its efficiency in image coding.

Notations: Throughout this paper, a special matrix is the reversal matrix $\mathbf{J}$.

2. REVIEW

2.1 Existing Lattice Structure

Fig. 1(a) shows a typical M-channel maximally decimated filter bank, where $H_i(z)$ and $F_i(z)$ are the k-th analysis and synthesis subband filters. The polyphase implementation of the filter bank is illustrated in Fig. 1(b), where $\mathbf{E}(z)$ and $\mathbf{R}(z)$ are, respectively, the type-I analysis polyphase matrix and the type-II synthesis polyphase matrix [3].

First, we consider an M-channel linear-phase perfect reconstruction filter bank with each filter length KM. The analysis polyphase matrix $\mathbf{E}(z)$ can always be factored as

$$\mathbf{E}(z) = \mathbf{G}_{K-1}(z)\mathbf{G}_{K-2}(z)\cdots\mathbf{G}_1(z)\mathbf{E}_0, \qquad (1)$$

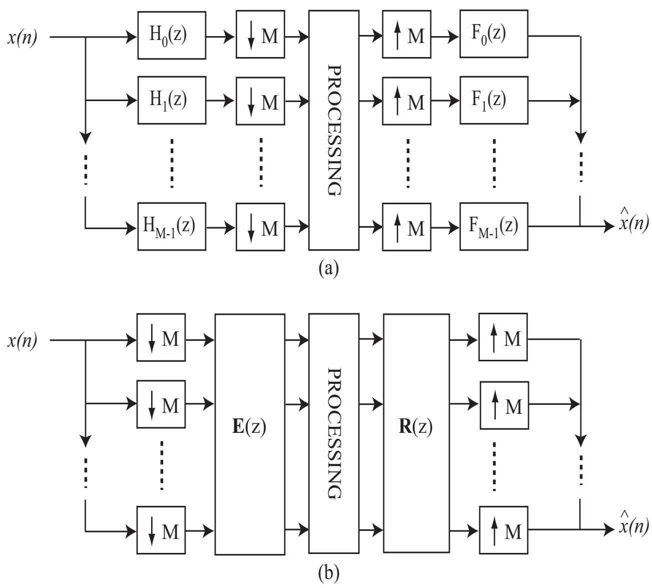


Figure 1: M-channel maximally decimated filter bank. (a) Basic structure, (b) Polyphase representation.

where $\mathbf{E}_0$ and $\mathbf{G}_i(z)$ are $M \times M$ nonsingular matrices [3, 4]. In addition, the filter bank can achieve perfect reconstruction, if and only if the synthesis bank $\mathbf{R}(z)$ can be chosen as

$$\mathbf{R}(z) = z^{-(K-1)}\mathbf{E}_0^{-1}\mathbf{G}_1^{-1}(z)\mathbf{G}_2^{-1}(z)\cdots\mathbf{G}_{K-1}^{-1}(z). \qquad (2)$$

When M is even, each matrix of (1) is represented as follows:

$$\begin{array}{c} \mathbf{G}_i(z) = \mathbf{\Phi}_i\mathbf{W}\mathbf{\Lambda}(z)\mathbf{W} \\ \mathbf{E}_0 = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{U}_0 & \mathbf{U}_0\mathbf{J} \\ \mathbf{V}_0 & \mathbf{V}_0\mathbf{J} \end{bmatrix}, \end{array} \qquad (3)$$

where $\mathbf{\Phi}_i = diag(\mathbf{U}_i, \mathbf{V}_i)$ and

$$\mathbf{W} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{I}_{\frac{M}{2}} & \mathbf{I}_{\frac{M}{2}} \\ \mathbf{I}_{\frac{M}{2}} & -\mathbf{I}_{\frac{M}{2}} \end{bmatrix}, \mathbf{\Lambda}(z) = \begin{bmatrix} \mathbf{I}_{\frac{M}{2}} & \mathbf{0}_{\frac{M}{2}} \\ \mathbf{0}_{\frac{M}{2}} & z^{-1}\mathbf{I}_{\frac{M}{2}} \end{bmatrix}.$$

If $\mathbf{U}_i$ and $\mathbf{V}_i$ which have the size $M/2 \times M/2$ are unitary matrices and nonsingular matrices, the filter bank is a linear-phase paraunitary filter bank (LPPUFB) and a linear-phase biorthogonal filter bank (LPBOFB), respectively.

Substituting (3) into (1), the left tail end of the LPFB coefficient matrix is shown as

$$\mathbf{P}_K = \alpha \begin{bmatrix} \mathbf{U}_{K-1} \prod_{i=K-2}^{0} (\mathbf{U}_i - \mathbf{V}_i) & \cdots \\ \mathbf{V}_{K-1} \prod_{i=K-2}^{0} (\mathbf{U}_i - \mathbf{V}_i) & \cdots \end{bmatrix} \quad (4)$$

where $\alpha = (1/\sqrt{2})^{2K-1}$ and $\mathbf{P}_K$ has the size $M \times KM$. The k-th row of (4) corresponds to the impulse response $h_k(n)$ of the analysis filter $H_k(z)$.

2.2 ULLPPUFB

In the past, LPPUFB whose N filters have length KM and $(M-N)$ filters have length $(K-1)M$ has been proposed [5]. When N is even, the structure in [5] has to keep the following condition

$$\mathbf{U}_{K-1} \prod_{i=K-2}^{0} (\mathbf{U}_i - \mathbf{V}_i) = \begin{bmatrix} \mathbf{X}_{\frac{N}{2} \times \frac{M}{2}} \\ \mathbf{0}_{\frac{M-N}{2} \times \frac{N}{2}} \end{bmatrix}. \quad (5)$$

If the product is defined as $\mathbf{Y} = \mathbf{U}_{K-1} \prod_{K-2}^{1} (\mathbf{U}_i - \mathbf{V}_i)$, the condition is to determine $\mathbf{U}_0$ and $\mathbf{V}_0$ such that $\mathbf{Y}(\mathbf{U}_0 - \mathbf{V}_0)$ has the desired form. When $\mathbf{U}_{K-1}$, $\mathbf{B}$, $\mathbf{U}_i$ and $\mathbf{V}_i (i = 1, 2, \cdots, K-2)$ are defined as arbitrary $M/2 \times M/2$ unitary matrices, ULLPPUFB can be realized by choosing $\mathbf{U}_0$, $\mathbf{V}_0$ and $\mathbf{V}_{K-1}$ as

$$\begin{aligned} \mathbf{U}_0 &= \mathbf{T} \begin{bmatrix} \mathbf{Q}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_3 \end{bmatrix} \mathbf{B} \\ \mathbf{V}_0 &= \mathbf{T} \begin{bmatrix} \mathbf{Q}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_3 \end{bmatrix} \mathbf{B} \\ \mathbf{V}_{K-1} &= \begin{bmatrix} \mathbf{Q}_4 & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_5 \end{bmatrix} \mathbf{U}_{K-1} \end{aligned} \quad (6)$$

where $\mathbf{T}$ is a unitary matrix calculated from $\mathbf{Y}$. Additionally, $\mathbf{Q}_1$, $\mathbf{Q}_2$ and $\mathbf{Q}_4$ are $N/2 \times N/2$ unitary matrices and $\mathbf{Q}_3$ and $\mathbf{Q}_5$ are $(M-N)/2 \times (M-N)/2$ unitary matrices, respectively. However, these parameters are redundant.

2.3 LPFB without redundancy

We consider the polyphase matrix $\mathbf{E}(z)$ by substituting (3) into (1). (1) is rewritten as

$$\mathbf{E}(z) = \begin{bmatrix} \mathbf{U}_{K-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_{K-1} \end{bmatrix} \mathbf{W} \mathbf{\Lambda}(z) \mathbf{W} \begin{bmatrix} \mathbf{U}_{K-2} & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_{K-2} \end{bmatrix} \cdots, \quad (7)$$

though this structure is redundant. In [7], the polyphase matrix of the LPFB without redundancy is represented as

$$\begin{aligned} \check{\mathbf{E}}(z) &= \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \check{\mathbf{V}}_{K-1} \end{bmatrix} \mathbf{W} \mathbf{\Lambda}(z) \mathbf{W} \\ &\times \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \check{\mathbf{V}}_{K-2} \end{bmatrix} \cdots \begin{bmatrix} \mathbf{U}_0 & \mathbf{U}_0 \mathbf{J} \\ \mathbf{V}_0 & \mathbf{V}_0 \mathbf{J} \end{bmatrix} \end{aligned} \quad (8)$$

where $\check{\mathbf{V}}_{K-1} = \mathbf{V}_{K-1} \mathbf{U}_{K-1}^{-1}$ and $\check{\mathbf{V}}_{K-2} = \mathbf{U}_{K-1} \mathbf{V}_{K-2}$. Thus, the new building block contains one-half of free invertible matrices as those of $\mathbf{G}_i(z)$. Besides, $\check{\mathbf{E}}(z)$ completely keeps the property of $\mathbf{E}(z)$. However, this structure can not be directly used by the traditional ULLPFBs. Therefore, we propose the new structure of unequal-length linear-phase filter banks from the viewpoint of eliminating redundancy by using this simplified lattice structure. For simplicity, we denote $\check{\mathbf{V}}_i = \mathbf{V}_i$ hereafter.

3. DESIGN OF ULLPFB WITHOUT REDUNDANCY

In this section, we present the condition for ULLPFBs without redundancy.

We consider an M-channel LPFB which is consisted of N filters with length KM and $(M-N)$ filters with length $(K-1)M$ (M:even, K: positive integer, N: even and $M \geq N$). Now, the left tail end of (8) is rewritten as

$$\check{\mathbf{P}}_K = \alpha \begin{bmatrix} \mathbf{P}(\mathbf{U}_0 - \mathbf{V}_0) & \cdots \\ \mathbf{V}_{K-1} \mathbf{P}(\mathbf{U}_0 - \mathbf{V}_0) & \cdots \end{bmatrix}, \quad (9)$$

where $\mathbf{P} = \prod_{i=K-2}^{1} (\mathbf{I} - \mathbf{V}_i)$. Consequently, (9) have to be

$$\begin{aligned} \mathbf{P}(\mathbf{U}_0 - \mathbf{V}_0) &= \begin{bmatrix} \mathbf{X}_{\frac{N}{2} \times \frac{M}{2}} \\ \mathbf{0}_{\frac{M-N}{2} \times \frac{M}{2}} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{\Delta}_{\frac{N}{2} \times \frac{N}{2}} & \mathbf{\Theta}_{\frac{N}{2} \times \frac{M-N}{2}} \\ \mathbf{0}_{\frac{M-N}{2} \times \frac{N}{2}} & \mathbf{\Xi}_{\frac{M-N}{2} \times \frac{M-N}{2}} \end{bmatrix} \begin{bmatrix} \mathbf{\Phi}_{\frac{N}{2} \times \frac{M}{2}} \\ \mathbf{0}_{\frac{M-N}{2} \times \frac{M}{2}} \end{bmatrix}. \end{aligned} \quad (10)$$

3.1 Paraunitary case

First, in the paraunitary case, we consider the relation between $\mathbf{U}_0$ and $\mathbf{V}_0$ in (10). In order to satisfy the condition in (10), we define $\mathbf{U}_0$ and $\mathbf{V}_0$ as

$$\mathbf{U}_0 = \mathbf{T}\mathbf{B}, \ \mathbf{V}_0 = \mathbf{T}\, diag(\mathbf{Q}_1, \mathbf{I}) \mathbf{B} \quad (11)$$

similarly to (6). $\mathbf{Q}_1$ and $\mathbf{B}$ are $N/2 \times N/2$ and $M/2 \times M/2$ unitary matrices, respectively. $\mathbf{T}$ is calculated from $\mathbf{P}$. Thus, substituting (11) into (10) gives the condition as follows:

$$\mathbf{P}\mathbf{T} = \begin{bmatrix} \mathbf{\Delta}_{\frac{N}{2} \times \frac{N}{2}} & \mathbf{\Theta}_{\frac{N}{2} \times \frac{M-N}{2}} \\ \mathbf{0}_{\frac{M-N}{2} \times \frac{N}{2}} & \mathbf{\Xi}_{\frac{M-N}{2} \times \frac{M-N}{2}} \end{bmatrix}. \quad (12)$$

In order to determine $\mathbf{T}$, we define

$$\mathbf{P} = \begin{bmatrix} \mathbf{C}_{\frac{N}{2} \times \frac{M}{2}} \\ \mathbf{D}_{\frac{M-N}{2} \times \frac{M}{2}} \end{bmatrix}. \quad (13)$$

Then, we orthonormalize $\mathbf{D}$ by the Gram-Schmidt algorithm and denote the orthonormalized matrix as $\mathbf{D}^{\perp}$. Moreover, we define the $M/2 \times M/2$ unitary matrix $\mathbf{T}^T$ that is calculated by setting $\mathbf{D}^{\perp}$ as the orthonormal basis:

$$\mathbf{T}^T = \begin{bmatrix} \mathbf{F}^{\perp}_{\frac{N}{2} \times \frac{M}{2}} \\ \mathbf{D}^{\perp}_{\frac{M-N}{2} \times \frac{M}{2}} \end{bmatrix}. \quad (14)$$

$\mathbf{D}$ is orthogonal to $\mathbf{F}^{\perp}$ in the above equation[1]. Consequently, we have

$$\begin{aligned} \mathbf{P}\mathbf{T} &= \begin{bmatrix} \mathbf{C}_{\frac{N}{2} \times \frac{M}{2}} \\ \mathbf{D}_{\frac{M-N}{2} \times \frac{M}{2}} \end{bmatrix} \begin{bmatrix} \mathbf{F}^{\perp T}_{\frac{M}{2} \times \frac{N}{2}} & \mathbf{D}^{\perp T}_{\frac{M}{2} \times \frac{M-N}{2}} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{\Delta}_{\frac{N}{2} \times \frac{N}{2}} & \mathbf{\Theta}_{\frac{N}{2} \times \frac{M-N}{2}} \\ \mathbf{0}_{\frac{M-N}{2} \times \frac{N}{2}} & \mathbf{\Xi}_{\frac{M-N}{2} \times \frac{M-N}{2}} \end{bmatrix}. \end{aligned} \quad (15)$$

Since $\mathbf{V}_{K-1} \mathbf{P}(\mathbf{U}_0 - \mathbf{V}_0)$ has to satisfy the condition in (10), $\mathbf{V}_{K-1}$ can be chosen as

$$\mathbf{V}_{K-1} = \begin{bmatrix} \mathbf{V}_{K-1,0} & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_{K-1,1} \end{bmatrix} \quad (16)$$

where $\mathbf{V}_{K-1,l}$ $(l = 0, 1)$ are $N/2 \times N/2$ and $(M-N)/2 \times (M-N)/2$ unitary matrices, respectively.

[1] $\mathbf{D}\mathbf{F}^{\perp T} = \mathbf{0}$

3.2 Biorthogonal case

In general, the ULLPBOFBs are classified into three types due to the biorthogonality. That is, the LPBOFB can have unequal-length filters at 1) the analysis bank, 2) the synthesis bank, and 3) both the analysis and the synthesis banks. Hence, we discuss the condition of each type.

3.2.1 Type-1: LPBOFB has unequal-length filters at the analysis bank

For sufficient condition in (10), $\mathbf{U}_0$ has to be identical to $\mathbf{V}_0$ at last $(M-N)/2$ rows. Therefore, nonsingular matrices $\mathbf{U}_0 = \mathbf{T}_a\hat{\mathbf{U}}_0$ and $\mathbf{V}_0 = \mathbf{T}_a\hat{\mathbf{V}}_0$ can be chosen as

$$\hat{\mathbf{V}}_0 = \begin{bmatrix} \multicolumn{2}{c}{\mathbf{Q}_{a\,\frac{N}{2}\times\frac{M}{2}}} \\ \mathbf{0}_{\frac{M-N}{2}\times\frac{N}{2}} & \mathbf{I}_{\frac{M-N}{2}\times\frac{M-N}{2}} \end{bmatrix} \hat{\mathbf{U}}_0 \qquad (17)$$

where $\hat{\mathbf{U}}_0$ is an $M/2 \times M/2$ nonsingular matrix. Then, the $M/2 \times M/2$ nonsingular matrix $\mathbf{T}_a$ is set to

$$\mathbf{PT}_a = \begin{bmatrix} \mathbf{\Delta}_{a\,\frac{N}{2}\times\frac{N}{2}} & \mathbf{\Theta}_{a\,\frac{N}{2}\times\frac{M-N}{2}} \\ \mathbf{0}_{\frac{M-N}{2}\times\frac{N}{2}} & \mathbf{\Xi}_{a\,\frac{M-N}{2}\times\frac{M-N}{2}} \end{bmatrix}. \qquad (18)$$

These conditions satisfy (10).

3.2.2 Type-2: LPBOFB has unequal-length filters at the synthesis bank

The condition that the synthesis bank has unequal filter length is indicated as

$$(\mathbf{U}_0^{-1} - \mathbf{V}_0^{-1})\mathbf{R} = \begin{bmatrix} \mathbf{\Phi}_{s\,\frac{M}{2}\times\frac{N}{2}} & \mathbf{0}_{\frac{M}{2}\times\frac{M-N}{2}} \end{bmatrix} \times \begin{bmatrix} \mathbf{\Delta}_{s\,\frac{N}{2}\times\frac{N}{2}} & \mathbf{0}_{\frac{N}{2}\times\frac{M-N}{2}} \\ \mathbf{\Theta}_{s\,\frac{M-N}{2}\times\frac{N}{2}} & \mathbf{\Xi}_{s\,\frac{M-N}{2}\times\frac{M-N}{2}} \end{bmatrix}, \qquad (19)$$

where $\mathbf{R} = \prod_{i=K-2}^{1}(\mathbf{I} - \mathbf{V}_i^{-1})$. In the same way at the analysis bank case, $\hat{\mathbf{U}}_0^{-1}$, $\hat{\mathbf{V}}_0^{-1}$ and $\mathbf{T}_s$ are

$$\hat{\mathbf{V}}_0^{-1} = \hat{\mathbf{U}}_0^{-1} \begin{bmatrix} \mathbf{Q}_{s\,\frac{M}{2}\times\frac{N}{2}} & \begin{matrix} \mathbf{0}_{\frac{N}{2}\times\frac{M-N}{2}} \\ \mathbf{I}_{\frac{M-N}{2}\times\frac{M-N}{2}} \end{matrix} \end{bmatrix}$$
$$\mathbf{T}_s\mathbf{R} = \begin{bmatrix} \mathbf{\Delta}_{s\,\frac{N}{2}\times\frac{N}{2}} & \mathbf{0}_{\frac{N}{2}\times\frac{M-N}{2}} \\ \mathbf{\Theta}_{s\,\frac{M-N}{2}\times\frac{N}{2}} & \mathbf{\Xi}_{s\,\frac{M-N}{2}\times\frac{M-N}{2}} \end{bmatrix}, \qquad (20)$$

where $\mathbf{U}_0^{-1} = \hat{\mathbf{U}}_0^{-1}\mathbf{T}_s$, $\mathbf{V}_0^{-1} = \hat{\mathbf{V}}_0^{-1}\mathbf{T}_s$ and $\hat{\mathbf{U}}_0^{-1}$ are $M/2 \times M/2$ nonsingular matrices, respectively.

3.2.3 Type-3: LPBOFB has unequal-length filters at both banks

In this case, the condition that the LPBOFB has unequal-length filters at both banks has to satisfy (10) and (19) simultaneously. However, such $\mathbf{T}_{as}$ which suffices (18) and (20) concurrently can not exist generally. Thus, we impose the restrictions into $\mathbf{V}_i$.

First, we consider the condition of the first block. We assume that the first matrix at the right side of (17) can be represented as

$$\begin{bmatrix} \mathbf{Q}_{a,0\,\frac{N}{2}\times\frac{N}{2}} & \mathbf{Q}_{a,1\,\frac{N}{2}\times\frac{M-N}{2}} \\ \mathbf{0}_{\frac{M-N}{2}\times\frac{N}{2}} & \mathbf{I}_{\frac{M-N}{2}\times\frac{M-N}{2}} \end{bmatrix},$$

this matrix is a block triangular matrix. Consequently, the inverse matrix of (17) is

$$\hat{\mathbf{V}}_0^{-1} = \hat{\mathbf{U}}_0^{-1} \begin{bmatrix} \mathbf{Q}_{a,0}{}^{-1} & -\mathbf{Q}_{a,0}{}^{-1}\mathbf{Q}_{a,1}{}^{-1} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}. \qquad (21)$$

Lena			
bpp	0.25	0.5	1.0
8×8 DCT	31.81	35.61	39.29
8×16 LOT	32.86	36.24	39.38
Proposed ULLPPUFB	32.89	36.32	39.63
Proposed type-1 ULLPBOFB	**33.30**	36.56	39.69
Proposed type-2 ULLPBOFB	33.29	**36.58**	**39.75**
Proposed type-3 ULLPBOFB	33.29	36.56	39.69

Boat			
bpp	0.25	0.5	1.0
8×8 DCT	28.61	31.91	35.61
8×16 LOT	29.05	32.34	35.79
Proposed ULLPPUFB	29.09	32.39	**35.87**
Proposed type-1 ULLPBOFB	**29.26**	**32.49**	35.86
Proposed type-2 ULLPBOFB	29.16	32.42	**35.87**
Proposed type-3 ULLPBOFB	**29.26**	**32.49**	**35.87**

Table 2: Comparison of PSNR[dB] (each ULLPFB is $M = 8$, $K = 2$ and $N = 6$).

The condition of (21) for being the structure of (20) is $\mathbf{Q}_{a,1}{}^{-1} = \mathbf{Q}_{a,1} = \mathbf{0}$. As a result, the structure of the first block is represented as

$$\mathbf{V}_0 = \begin{bmatrix} \mathbf{Q}_{a,0\,\frac{N}{2}\times\frac{N}{2}} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{\frac{M-N}{2}\times\frac{M-N}{2}} \end{bmatrix} \mathbf{U}_0. \qquad (22)$$

Next, we consider the form of the building block in order to suffice the condition. That is, $\mathbf{V}_i$ can be directly chosen parameters as

$$\mathbf{V}_i = \begin{bmatrix} \mathbf{\Delta}_{as\,\frac{N}{2}\times\frac{N}{2}} & \mathbf{0}_{\frac{N}{2}\times\frac{M-N}{2}} \\ \mathbf{0}_{\frac{M-N}{2}\times\frac{N}{2}} & \mathbf{\Xi}_{as\,\frac{M-N}{2}\times\frac{M-N}{2}} \end{bmatrix}. \qquad (23)$$

4. RESULT

In this section, we show several design examples of the proposed ULLPFBs and apply to image coding. Furthermore, we compare the proposed method with traditional ones.

The proposed ULLPFBs are optimized by the cost function which is a weighted linear combination of coding gain, DC leakage and stopband attenuation, all of which are well-known desired properties of FBs for image compression [4]. Table 1 shows the comparison of the coding gain and the number of free parameters. The proposed method indicates higher coding gain than all traditional ULLPFBs in spite of less free parameters. That is the reason that the proposed FBs converge to the optimal solution, while the traditional method may converge to the local solution owing to the huge number of parameters. Especially, when $K = 3$ and $N = 6$, the proposed type-3 (each bank has unequal-length filters) ULLPBOFB has 37.5% fewer parameters than the traditional ULLPBOFB whose analysis bank has unequal-length filters, despite the higher coding gain.

Table 2 shows the comparison of PSNR in image coding application for standard 512×512 images Lena and Boat. Each image is coded by the 6-level SPIHT [8]. The result shows that the proposed FBs have better PSNR than the traditional LOT [2] and DCT because the proposed FBs reduce not only the blocking effect but the ringing effect. That is an advantage of the reconstructed image quality mentioned in [5].

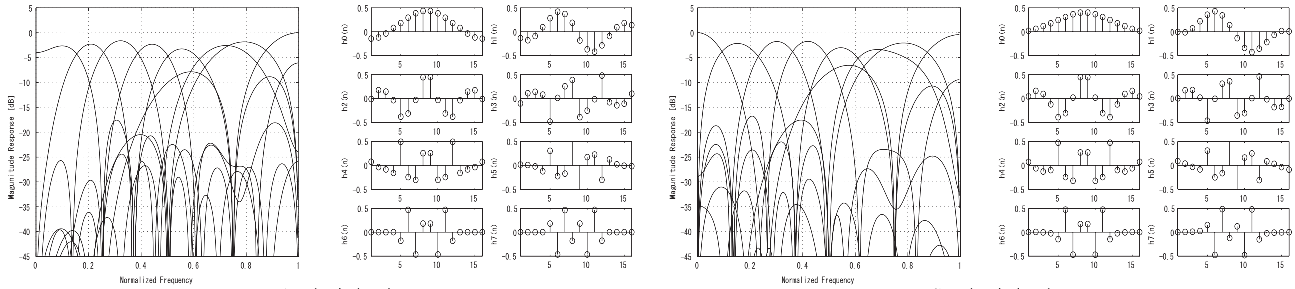
Fig. 2 shows the magnitude and impulse responses of the proposed $M = 8$, $K = 2$ ULLPBOFBs.

5. CONCLUSION

In this paper, we presented a new design method for ULLPFBs from the viewpoint of eliminating redundancy. The results show

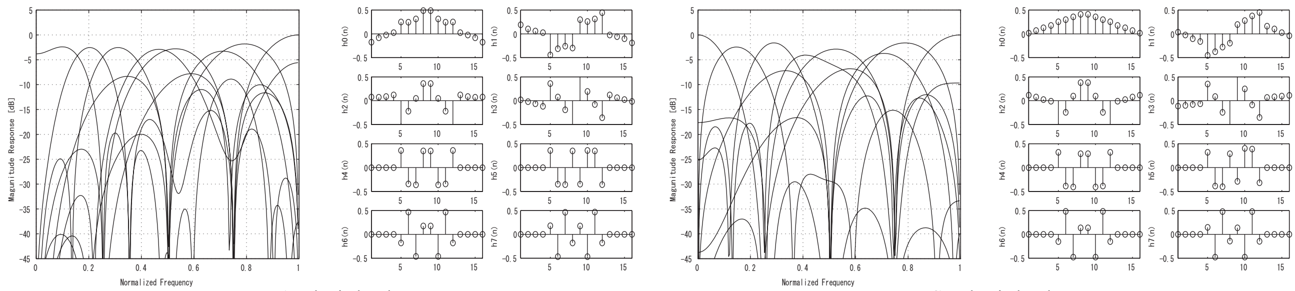
	Type	Filter length (long/ short)	N	Conventional [5] Coding gain	Conventional [5] FPs	Conventional [6] Coding gain	Conventional [6] FPs	Proposed Coding gain	Proposed FPs
Paraunitary case	-	$16/8$	4	8.977	17	8.988	11	8.988	9
		$16/8$	6	-	-	9.220	13	9.220	12
		$24/16$	6	-	-	9.367	21	9.367	18
Biorthogonal case	1	$16/8$	4	-	-	9.365	40	9.407	40
		$16/8$	6	-	-	9.509	46	9.593	44
		$24/16$	6	-	-	9.568	72	9.606	60
	2	$16/8$	4	-	-	-	-	9.561	40
		$16/8$	6	-	-	-	-	9.613	44
		$24/16$	6	-	-	-	-	9.625	60
	3	$16/8$	4	-	-	-	-	9.382	28
		$16/8$	6	-	-	-	-	9.590	35
		$24/16$	6	-	-	-	-	9.603	45

Table 1: Comparison of the coding gain [dB] and the number of free parameters (FPs) when $M = 8$.



Analysis bank Synthesis bank

(a)



Analysis bank Synthesis bank

(b)

Figure 2: Design examples of $M = 8$, $K = 2$ ULLPBOFBs: (a) type-1 ($N = 6$) and (b) type-3 ($N = 4$).

that ULLPFBs can be achieved by fewer parameters than traditional ones. Furthermore, the proposed ULLPFBs indicate higher coding gain in comparison with the traditional ULLPFBs. The proposed FBs' advantage of reducing both the blocking and the ringing effect can be verified by the comparison of PSNRs.

REFERENCES

[1] K. R. Rao and P. Yip, *Discrete Cosine Transform: Algorithms, Advantages, Applications,* New York: Academic, 1990.

[2] H. S. Malvar and D. H. Staelin, "The LOT: transform coding without blocking effects," *IEEE Trans. Signal Processing,* vol.37, pp.553–559, Oct. 1989.

[3] P. P. Vaidyanathan, *Multirate Systems and Filter Banks,* Englewood Cliffs, NJ: Prentice-Hall, 1993.

[4] G. Strang and T. Q. Nguyen, *Wavelets and Filter Banks,* Cambridge, MA: Wellesley-Cambridge, 1996.

[5] T. D. Tran, M. Ikehara, and T. Q. Nguyen, "Linear phase paraunitary filter bank with filters of different lengths and its application in image compression," *IEEE Trans. Signal Processing,* vol. 47, pp. 2730–2744, Oct. 1999.

[6] A. Saito, T. Urushibara, and M. Ikehara, "Design of generalized filter banks with unequal-length and its application of image coding," *IEICE Trans. Fundamentals,* vol. E87-A, pp. 1756–1764, Jul. 2004.

[7] L. Gan and K.-K. Ma, "A simplified lattice factorization for linear-phase perfect reconstruction filter bank," *IEEE Signal Processing Lett.,* vol. 8, pp. 207–209, Jul. 2001.

[8] A. Said and W. A. Pearlman, "A new, fast and efficient image codec based on set partitioning in hierarchical trees," *IEEE Trans. Circuits Syst. Video Technol.,* vol.6, pp.243–250, Jun. 1996.

[9] T. D. Tran, R. L. de Queiroz, and T. Q. Nguyen, "Linear phase perfect reconstruction filter bank: lattice structure, design, and application in image coding," *IEEE Trans. Signal Processing,* vol. 48, pp. 133-147, Jan. 2000.