A Non-Expansive Convolution for Nonlinear-Phase Paraunitary Filter Banks and Its Application to Image Coding

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***Abstract*— This paper proposes a new non-expansive convolution for nonlinear-phase paraunitary filter banks (NLPPUFBs). First, we present that any NLPPUFBs can be implemented by connecting several block transforms. Next, we show a signal extension method at the analysis bank by exploiting the characteristics of that structure. Furthermore, we prove that the signal can be reconstructed at the synthesis bank without any redundant signals. Finally, we apply the proposed extension to image coding to validate our method.**

I. Introduction

The lapped orthogonal transform (LOT) and its generalized version (GenLOT) play an important role in the transform coding and have been extensively studied [1], [9]. From a filter bank perspective, the LOT and the GenLOT are particular class of M-channel linear-phase paraunitary filter banks (LPPUFBs), where the length L of each filter is KM where K is a positive integer [2], [3]. They can avoid the blocking effect which is a shortcoming of the DCT [4], because they have long basis functions. However, they cause the problem that the total number of transform coefficients is more than the input signals by the signal convolution if the signal lengths are finite. This expansive effect is undesirable in image compression applications.

One simple non-expansive approach is to use the circular convolution for the finite length signals. Though this approach does not increase the amount of information to be coded, the periodic extension causes the signal discontinuities at the boundaries and requires more bits to code large transform coefficients in the high frequency bands.

The symmetric extension is an efficient method for linear-phase filter banks (LPFBs) to overcome the problem of the boundary distortion [5]. Since the symmetric extension improves the performance of image coding applications, most works in the field of filter bank systems are focused on LPFBs. However, the linear-phase property is the additional constraint on the filter design. In [6], it has been shown that the performance of stopband attenuation and coding gain of the LPPUFB is worse than the general (nonlinear-phase) PUFBs. Although the signal extension method for NLPPUFBs has been proposed [7], it increases the computational complexity because it has to calculate the extension signals at the analysis bank.

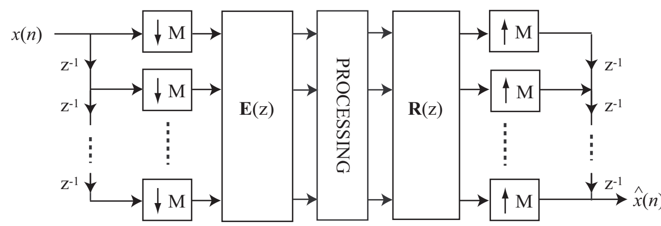

Fig. 1. The polyphase representation of an M-channel maximally decimated filter bank.

In order to find a simpler extension method, we focus on that any NLPPUFBs can be implemented by connecting several block transforms. In this paper, we propose a novel non-expansive convolution method for M-channel NLPPUFBs. With the proposed extension, it is shown that NLPPUFBs have superior coding performance to LPPUFBs.

II. Existing Lattice Structure

Fig. 1 shows a polyphase implementation of a typical M-channel maximally decimated filter bank [2], [3]. If an M-channel NLPPUFB has length KM, the analysis polyphase matrix $\mathbf{E}(z)$ is represented as [6]

$$\mathbf{E}(z) \quad \mathbf{G}_{K-1}(z)\mathbf{G}_{K-2}(z)\cdots\mathbf{G}_1(z)\mathbf{E}_0. \tag{1}$$

If M is even, each matrix of (1) is represented as follows:

$$\mathbf{G}_i(z) \quad \mathbf{P}_i\mathbf{W}_i\mathbf{\Lambda}(z)\mathbf{W}_i \tag{2}$$

where $\mathbf{P}_i \quad diag(\mathbf{U}_i, \mathbf{V}_i)$ and

$$\mathbf{W}_i \quad \begin{bmatrix} \mathbf{C}_i & \mathbf{S}_i \\ \mathbf{S}_i & -\mathbf{C}_i \end{bmatrix}, \mathbf{\Lambda}(z) \quad \begin{bmatrix} \mathbf{I}_{\frac{M}{2}} & \mathbf{0}_{\frac{M}{2}} \\ \mathbf{0}_{\frac{M}{2}} & z^{-1}\mathbf{I}_{\frac{M}{2}} \end{bmatrix}.$$

$\mathbf{E}_0$ is an $M \times M$ unitary matrix, $\mathbf{U}_i$ and $\mathbf{V}_i$ are $M/ \ \times M/$ unitary matrices and $\mathbf{C}_i$ and $\mathbf{S}_i$ are $M/ \ \times M/$ diagonal matrices as $\mathbf{C}_i \quad diag(\quad \alpha_{ll})$ and $\mathbf{S}_i \quad diag(\quad \alpha_{ll})$ $(l$

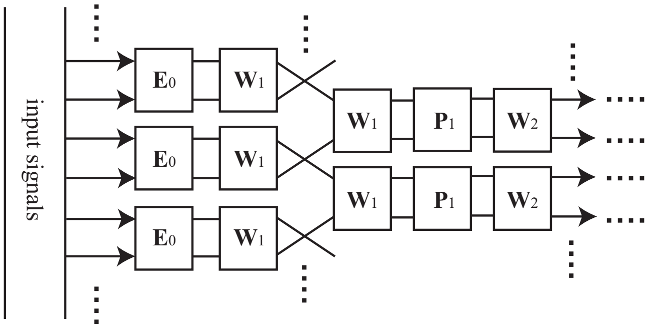

Fig. 2. A form of an M-channel NLPPUFB connecting several block transforms.

, , $\cdots, M/$), respectively. Furthermore, in the real coefficients case, the synthesis polyphase matrix $\mathbf{R}(z)$ is shown as

$$\mathbf{R}(z) \quad z^{-(K-1)}\mathbf{E}_0^T\mathbf{G}_1^T(z)\mathbf{G}_2^T(z)\cdots\mathbf{G}_{K-1}^T(z) \qquad (3)$$

from the orthogonality of $\mathbf{E}(z)$.

III. A Non-expansive Convolution for NLPPUFBs

When FBs are applied to image coding, the signal increasing problem occurs at boundaries of images due to the signal convolution. The LPFB can avoid this problem by utilizing the symmetric extension. In contrast, the NLP one can not use the symmetric extension. Hence, we present a signal extension/reconstruction method using the lattice structure of the NLPPUFB in this section.

Attention: For simplicity, we represent each line with/without arrow in Figs. 2–4 as a signal vector consisted of $M/$ signals. We also denote signal $\mathbf{p}$ in the synthesis bank which corresponds to signal $\mathbf{p}$ in the analysis bank.

A. Signal extension at the analysis bank

We consider an M-channel NLPPUFB with its length KM (M: even). In (2), $\mathbf{\Lambda}(z)$ makes $M/$ signals delay and their delayed signals are input to next building block. Therefore, an NLPPUFB is represented in Fig. 2 by connecting several $M \times M$ block transforms. $M/$ samples cross between each $\mathbf{W}_i$ due to the delay elements.

Fig. 3(a) shows the upper image boundary of Fig. 2 when K . From that figure, $M/$ signals have to be extended at the upper and the lower image boundaries, respectively. Consequently, the whole input signal has to be extended by M signals. Now we extend signals by using $\mathbf{J}_{\frac{M}{2}}$ for smooth extension.

With the reverse operation, the signal has to be reconstructed at the synthesis bank. However, transmitted signals are only $\mathbf{y}_n$ $(n \quad , \ , \ ,\cdots)$. Hence, we have to solve the "non-expansive" problem. That is, we have to find a solution to reconstruct $\mathbf{x}_0$ from $\mathbf{z}_n$ which are calculated from $\mathbf{y}_n$ easily. This problem is solved in the next subsection.

We also show the upper image boundary when K and in Fig. 3(b) and (c), respectively. This is similar to K . After all, the input signal has to be extended by $(K- \ )M/$ signals at each image boundary, by using $\mathbf{J}_{\frac{(K-1)M}{2}}$ for smooth extension. Each block which has the size $M \times M$ in Fig. 3 is represented as follows:

$$K \qquad \mathbf{A} \quad \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \quad \mathbf{W}_1\mathbf{E}_0$$

$$K \quad \begin{cases} \mathbf{B} \quad \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix} \quad \mathbf{W}_1\mathbf{E}_0 \\ \mathbf{C} \quad \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix} \quad \mathbf{W}_2\mathbf{P}_1\mathbf{W}_1 \end{cases}$$

$$K \quad \begin{cases} \mathbf{D} \quad \begin{bmatrix} \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{D}_{21} & \mathbf{D}_{22} \end{bmatrix} \quad \mathbf{W}_1\mathbf{E}_0 \\ \mathbf{F} \quad \begin{bmatrix} \mathbf{F}_{11} & \mathbf{F}_{12} \\ \mathbf{F}_{21} & \mathbf{F}_{22} \end{bmatrix} \quad \mathbf{W}_2\mathbf{P}_1\mathbf{W}_1 \\ \mathbf{H} \quad \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} \quad \mathbf{W}_3\mathbf{P}_2\mathbf{W}_2 \end{cases}$$

where each submatrix has the size $M/ \ \times M/$.

B. Signal reconstruction at the synthesis bank

In this subsection, our purpose is to reconstruct $\mathbf{x}_n$ from $\mathbf{z}_n$, $\mathbf{a}_n$ and $\mathbf{b}_n$ in Fig. 3. We show the signal reconstruction method for some K.

Hereafter, we consider only the upper image boundary because the signal can be reconstructed by the same way at the lower image boundary.

1) K : In Fig. 3(a), we have

$$\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \end{bmatrix} \begin{bmatrix} \mathbf{J}\mathbf{x}_0 \\ \mathbf{x}_0 \end{bmatrix} \quad \mathbf{z}_0. \qquad (4)$$

The problem is to reconstruct $\mathbf{x}_0$ from $\mathbf{z}_0$ which is transmitted to the synthesis bank. From (4), $\mathbf{x}_0$ is represented as the solution of the matrix equation $(\mathbf{A}_{11}\mathbf{J} + \mathbf{A}_{12})\mathbf{x}_0 \quad \mathbf{z}_0$. Therefore, we get

$$\mathbf{x}_0 \quad (\mathbf{A}_{11}\mathbf{J} + \mathbf{A}_{12})^{-1}\mathbf{z}_0. \qquad (5)$$

This procedure is shown in Fig. 4(a). In this case, the matrix $(\mathbf{A}_{11}\mathbf{J} + \mathbf{A}_{12})$ has to be nonsingular. In the next section, we make consideration of this condition to design the NLPPUFB.

2) K : The problem is to calculate $\mathbf{a}_1$ in Fig. 3(b). In this case, we have

$$\mathbf{a}_0 \quad \mathbf{B}_{11}\mathbf{J}\mathbf{x}_1 + \mathbf{B}_{12}\mathbf{J}\mathbf{x}_0 \quad \begin{bmatrix} \mathbf{B}_{11}\mathbf{J} & \mathbf{B}_{12}\mathbf{J} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_0 \end{bmatrix} \qquad (6)$$

and

$$\begin{bmatrix} \mathbf{x}_0 \\ \mathbf{x}_1 \end{bmatrix} \quad \begin{bmatrix} \mathbf{B}_{11}^T & \mathbf{B}_{21}^T \\ \mathbf{B}_{12}^T & \mathbf{B}_{22}^T \end{bmatrix} \begin{bmatrix} \mathbf{a}_2 \\ \mathbf{a}_1 \end{bmatrix}. \qquad (7)$$

Substitute (7) into (6), then (6) is rewritten as

$$\mathbf{a}_0 \quad \begin{bmatrix} \mathbf{B}_{12}\mathbf{J} & \mathbf{B}_{11}\mathbf{J} \end{bmatrix} \begin{bmatrix} \mathbf{B}_{11}^T & \mathbf{B}_{21}^T \\ \mathbf{B}_{12}^T & \mathbf{B}_{22}^T \end{bmatrix} \begin{bmatrix} \mathbf{a}_2 \\ \mathbf{a}_1 \end{bmatrix}. \qquad (8)$$

Furthermore, we also have

$$\mathbf{z}_0 \quad \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \end{bmatrix} \begin{bmatrix} \mathbf{a}_0 \\ \mathbf{a}_1 \end{bmatrix} \qquad (9)$$

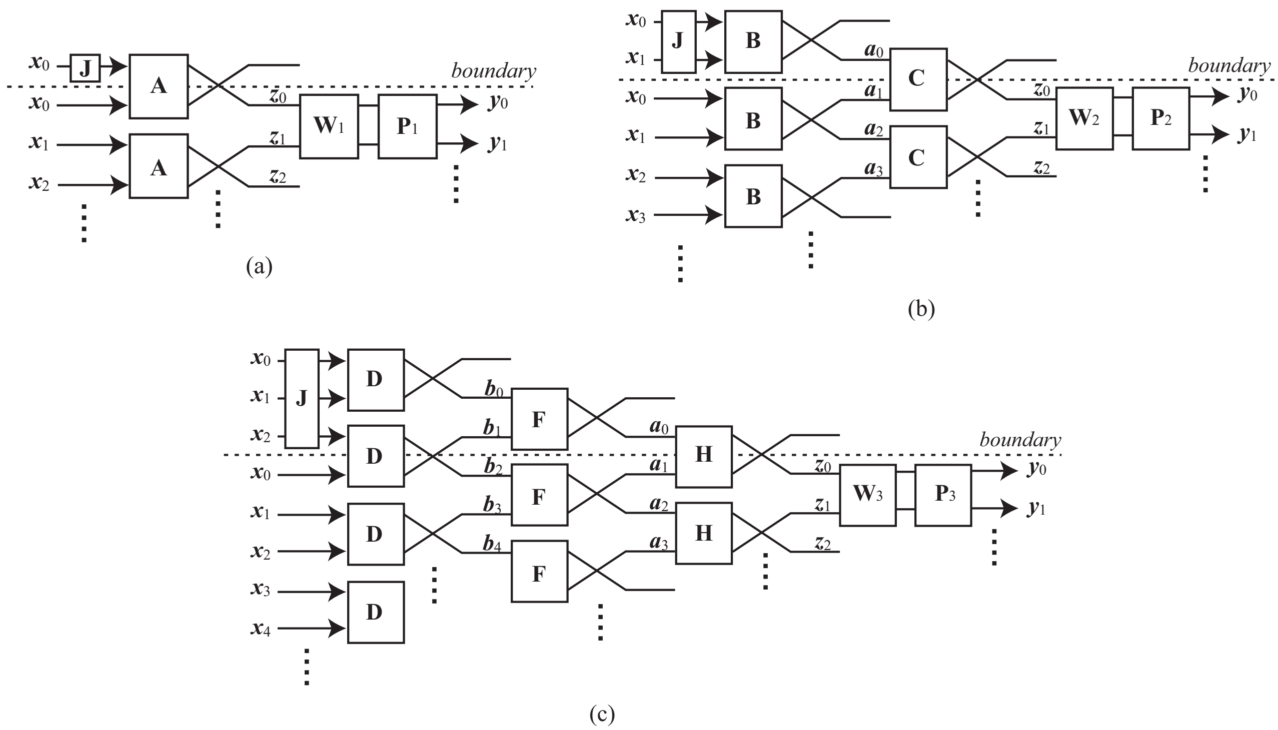

Fig. 3. The proposed extension for NLPPUFBs (dotted lines represent the upper image boundaries): (a) $K = 2$, (b) $K = 3$ and (c) $K = 4$.

from Fig. 3(b). Hence $\mathbf{a}_1$ can be calculated as the solution of the simultaneous matrix equation with two unknowns $\mathbf{a}_0$ and $\mathbf{a}_1$ represented as (8) and (9). Finally we get

$$\mathbf{a}_1 \quad (\mathbf{C}_{11}\mathbf{\Delta} + \mathbf{C}_{12})^{-1}(\mathbf{z}_0 - \mathbf{C}_{11}\mathbf{\Xi}\mathbf{a}_2) \tag{10}$$

if $(\mathbf{C}_{11}\mathbf{\Delta} + \mathbf{C}_{12})$ is nonsingular. We make consideration of this condition to design the NLPPUFB in the next section as well as K . In (10), we define

$$\begin{aligned} \mathbf{\Delta} &\quad (\mathbf{B}_{12}\mathbf{J}\mathbf{B}_{11}^T + \mathbf{B}_{11}\mathbf{J}\mathbf{B}_{12}^T) \\ \mathbf{\Xi} &\quad (\mathbf{B}_{12}\mathbf{J}\mathbf{B}_{21}^T + \mathbf{B}_{11}\mathbf{J}\mathbf{B}_{22}^T). \end{aligned}$$

The reconstruction procedure is illustrated in Fig. 4(b).

3) $K \geq$: In any K, the signal can be reconstructed as the solution of the simultaneous matrix equation with $(K-\ )$ unknowns as well as the previous cases.

As an example, the solution in K is shown as follows:

$$\begin{aligned} \mathbf{a}_1 &\quad (\mathbf{H}_{11}\mathbf{F}_{11}\mathbf{\Upsilon}\mathbf{F}_{22}^T + \mathbf{H}_{11}\mathbf{F}_{12}\mathbf{\Phi}\mathbf{F}_{21}^T + \mathbf{H}_{12})^{-1} \\ &\times(\mathbf{z}_0 - \mathbf{H}_{11}(\mathbf{F}_{11}\mathbf{\Upsilon}\mathbf{F}_{12}^T + \mathbf{F}_{12}\mathbf{\Phi}\mathbf{F}_{11}^T)\mathbf{a}_2 - \mathbf{H}_{11}\mathbf{F}_{11}\mathbf{\Psi}\mathbf{b}_4). \end{aligned} \tag{11}$$

The condition is that $(\mathbf{H}_{11}\mathbf{F}_{11}\mathbf{\Upsilon}\mathbf{F}_{22}^T + \mathbf{H}_{11}\mathbf{F}_{12}\mathbf{\Phi}\mathbf{F}_{21}^T + \mathbf{H}_{12})$ is nonsingular, and we define

$$\begin{aligned} \mathbf{\Upsilon} &\quad \mathbf{D}_{12}\mathbf{J}\mathbf{D}_{21}^T + \mathbf{D}_{11}\mathbf{J}\mathbf{D}_{22}^T \\ \mathbf{\Phi} &\quad (\mathbf{D}_{21}\mathbf{J} + \mathbf{D}_{22})(\mathbf{D}_{11}\mathbf{J} + \mathbf{D}_{12})^{-1} \\ \mathbf{\Psi} &\quad \mathbf{D}_{12}\mathbf{J}\mathbf{D}_{11}^T + \mathbf{D}_{11}\mathbf{J}\mathbf{D}_{12}^T. \end{aligned}$$

It is similar to K to reconstruct the signal from the next block transform.

IV. Results

In this section, we design some NLPPUFBs and apply these to image coding with the proposed extension. We compare our proposed method with the traditional DCT, LOT with the symmetric extension and NLPPUFB with the traditional extension [7].

The cost function to design NLPPUFBs is defined as the linear combination of coding gain, stopband attenuation, DC leakage and symmetric property of filters. First three motivations are quite popular [2], and symmetric property of filters is defined as

$$C_{sym} \quad \sum_{i=0}^{M-1} \sum_{n=0}^{\frac{MK}{2}-1} (h_i(n) - sh_i(MK - \ - n)), \tag{12}$$

where $h_i(n)$s are the impulse responses of the $H_i(z)$. Additionally, we set s in the channel with even index and s $-$ in the channel with odd index. This restriction is required to satisfy the condition that the matrices in (5) and (10) are nonsingular. We designed two 8-channel NLPPUFBs with its length 16 (K) and 24 (K). Their magnitude and impulse responses are shown in Fig. 5.

Table I shows the comparison of PSNRs of the reconstructed images. Each image is coded by 6-level SPIHT [8]. As a result, the proposed method indicates higher PSNR in all images. The $\times$ NLPPUFB with the proposed extension signifies 0.3–0.4 dB higher PSNR than the $\times$ LOT [9] with the symmetric extension in Lena image (especially 0.43 dB higher at 1.0 bpp). This result represents the proposed method can extend the

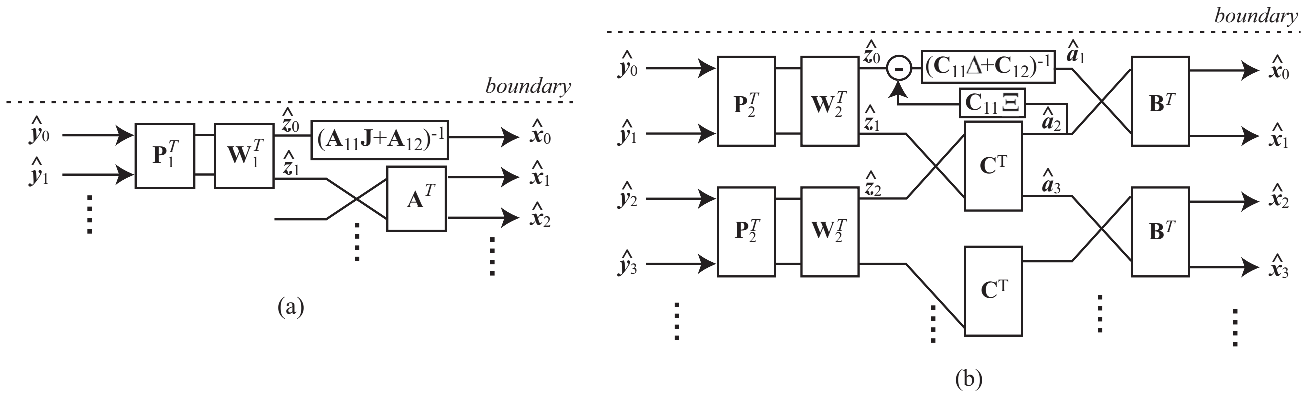

Fig. 4. The signal reconstruction method at the synthesis bank: (a) $K = 2$ and (b) $K = 3$.

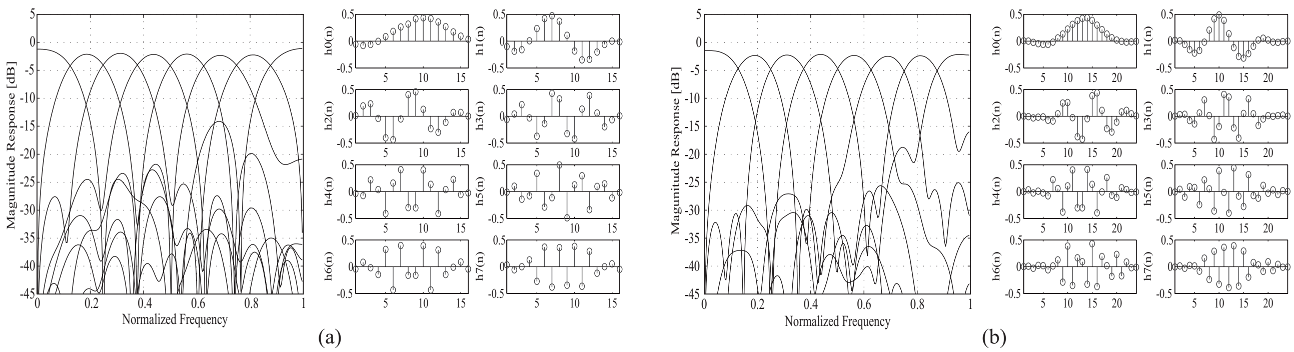

Fig. 5. The magnitude and impulse responses of the NLPPUFBs (the analysis banks): (a) 8×16 and (b) 8×24.

TABLE I
THE COMPARISON OF PSNR [DB] (SYM.: THE SYMMERTIC EXTENSION, PROP.: THE PROPOSED EXTENSION).

Image	Lena			Goldhill			Pepper		
	bpp			bpp			bpp		
Transform/ extension	0.25	0.5	1.0	0.25	0.5	1.0	0.25	0.5	1.0
8×8 DCT	31.81	35.61	39.29	29.34	32.00	35.47	31.39	34.54	37.13
8×16 LOT/ Sym.	32.86	36.24	39.38	29.81	32.44	35.78	32.14	34.79	37.11
8×24 GenLOT/ Sym.	33.08	36.45	39.77	29.87	32.49	35.86	32.31	34.91	37.30
8×16 NLPPUFB/ [7]	33.17	36.06	39.35	29.73	32.46	35.86	31.55	34.27	36.82
8×24 NLPPUFB/ [7]	32.59	36.13	39.33	29.84	32.49	35.85	31.61	34.20	36.72
8×16 NLPPUFB/ Prop.	33.21	36.55	39.81	29.95	**32.57**	**35.91**	32.38	35.02	**37.38**
8×24 NLPPUFB/ Prop.	**33.36**	**36.66**	**39.86**	**29.96**	32.56	35.90	**32.54**	**35.05**	37.36

signal without spoiling the property of the NLPPUFB, which has higher coding gain.

V. CONCLUSION

This paper proposed a new non-expansive convolution method for NLPPUFBs to apply to image coding. We presented that the signal can be reconstructed at the synthesis bank without any redundant signals. In the result of image coding, the proposed method has higher PSNR than the traditional ones. Our future work is to find more effective extension matrices to improve the coding performance.

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