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LIFTING-BASED PARAUNITARY FILTERBANKS FOR LOSSY/LOSSLESS IMAGE CODING

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ABSTRACT

This paper introduces one of the image transform methods using M-channel paraunitary filterbanks (PUFBs) based on Householder matirx. First, redundant parameters of PUFB are eliminated by using the fact that they can be factorized into Givens rotation matices. Next, we propose an eliminating redundant parameters method based on Householder matrix using relationship between Givens rotation and Householder matrices. In addition, PUFBs are factorized into the lifting structure for lossless image coding, and we call them as lifting-based PUFBs (LBPUFBs). LBPUFBs based on Householder matrix have less number of rounding operators than Givens rotation matrix version, since proposed structure is efficiency for lossless image coding. Finally, we show some exsamples to validate our method in lossy/lossless image coding.

Index Terms — Paraunitary filterbank, householder matrix, redundant parameters, lifting structure, lossless image coding.

1. INTRODUCTION

Filterbanks (FBs) have been found many applications such as speech, audio and video compression, statistical signal processing, discrete multitone modulation and channel equalization [2, 4]. Fig. 1 shows an M-channel maximally decimated FB, where $H_k(z)$ and $F_k(z)$ are the k-th (for $k = 0, \cdots, M-1$) analysis and synthesis filter, respectively. Also Fig. 2 shows a polyphase structure of FB. The analysis and synthesis filters are represented by using the polyphase matrices $\mathbf{E}(z)$ and $\mathbf{R}(z)$ as follows:

$$\begin{aligned} [H_0(z)\, H_1(z) \cdots H_{M-1}(z)]^T &= \mathbf{E}(z^M)\mathbf{e}(z)^T \\ [F_0(z)\, F_1(z) \cdots F_{M-1}(z)] &= \mathbf{e}(z)\mathbf{R}(z^M) \end{aligned} \quad (1)$$

where $\mathbf{e}(z) = [1, z^{-1}, \cdots, z^{-(M-1)}]$. If $\mathbf{E}^\dagger(z^{-1})\mathbf{E}(z) = \mathbf{I}$ and $\mathbf{R}(z) = \mathbf{E}^\dagger(z^{-1})$, where $.^\dagger$ stands for the conjugate transpose, the FBs are called paraunitary filterbanks (PUFBs). PUFBs are efficiently designed and implemented by the lattice structure. Recently, the complete and minimal lattice structure of PUFBs has been proposed by Gao *et al.* [7]. However its structure is still redundant, thus a simpler structure is developed [10]. Though PUFBs present good coding results for lossy image coding, they are not applied to lossless one.

In this paper, by reducing the redundant parameters in Householder matrix, we derive a novel lifting structure for PUFBs which has less implementation costs similar to [10]. The proposed PUFBs are not only applied to lossless image coding but lossy image coding. Our PUFBs with lifting structures are called Lifting-Based PUFBs (LBPUFBs). We show that the LBPUFBs is more superior results than 9/7-tap and 5/3-tap wavelet transforms (WTs) adopted in JPEG2000 [8].

Notations: $\mathbf{I}$, $\mathbf{M}^{(N)}$ and $\mathbf{M}^\dagger$ are the indetity matrix, the $N \times N$ square matrix and the conjugate transpose of a matrix, respectively. And $\| \mathbf{m} \| = \sqrt{\sum_{i=1}^{N} | m_i |^2}$ where $\mathbf{m} = [m_1, m_2, \cdots, m_N]^T$.

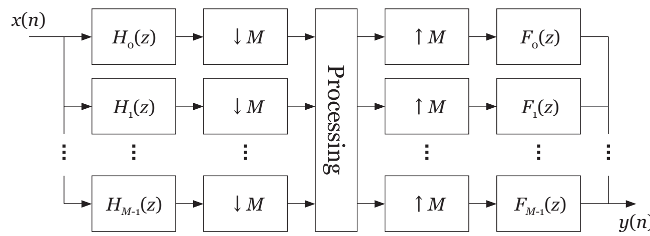

Figure 1: An M-channel filterbank.

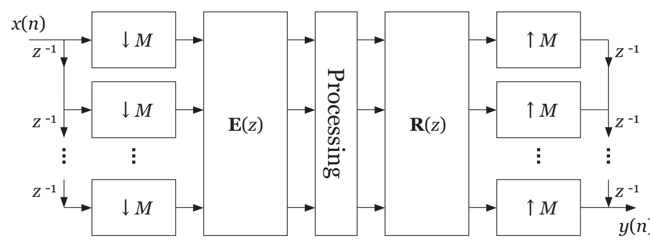

Figure 2: A polyphase structure of a filterbank.

2. PARAUNITARY FILTERBANKS BASED ON GIVENS ROTATION MATRIX

2.1 Lattice Structure

In this paper, we consider PUFBs where the number of channels is M (even), all filter lengths are MK ($K \in \mathbb{N}$), and we set $L = K - 1$.

The polyphase matrix $\mathbf{E}(z)$ of the PUFBs is represented as [7]

$$\mathbf{E}(z) = \mathbf{X}_L \mathbf{\Lambda}_L(z) \cdots \mathbf{X}_1(z) \mathbf{\Lambda}_1(z) \mathbf{X}_0 \quad (2)$$

where

$$\mathbf{\Lambda}_k(z) = \begin{bmatrix} \mathbf{I}^{(M-\gamma_k)} & \mathbf{0} \\ \mathbf{0} & z^{-1}\mathbf{I}^{(\gamma_k)} \end{bmatrix} \quad (3)$$

and $\mathbf{X}_k$s are $M \times M$ arbitrary orthogonal matrices. Although γ_k is arbitrary integer $1 \le \gamma_k < M$, in this paper we set $\gamma_k = M/2$ for simplicity. Thus $\mathbf{\Lambda}_k(z)$ is denoted as $\mathbf{\Lambda}(z)$.

2.2 Givens Rotation Matrix Factorization

An $M \times M$ orthogonal matrix $\mathbf{X}$ is factorized into a product of $M(M-1)/2$ rotation angles $\mathbf{\Theta}_{i,j}$ [1]

$$\mathbf{X} = \prod_{i=1}^{M-1} \prod_{j=i+1}^{M} \mathbf{\Theta}_{i,j} \quad (4)$$

where

$$\left[\mathbf{\Theta}_{i,j}\right]_{k,l} = \begin{cases} 1 & : k = l \neq i \text{ or } j \\ \cos\theta_{i,j} & : k = l = i \text{ or } j \\ -\sin\theta_{i,j} & : k = i \text{ and } l = j \\ \sin\theta_{i,j} & : k = j \text{ and } l = i \\ 0 & : otherwise \end{cases} . \tag{5}$$

Here notes that an order of the Givens rotation angles is arbitrary. This means that an orthogonal matrix is represented as many kinds of structures whose the order of Givens rotation angles are different.

2.3 Simple Structure for PUFBs

In [10], it is shown a lattice structure which has less implementation costs than that in [7]. The lattice structure in [10] is represented as

$$\mathbf{E}(z) = \left(\prod_{k=L}^{1} \widetilde{\mathbf{X}}_k \mathbf{\Lambda}(z)\right) \mathbf{X}_0 \tag{6}$$

where

$$\widetilde{\mathbf{X}}_k = \prod_{i=1}^{M/2} \prod_{j=M/2+1}^{M} \mathbf{\Theta}_{i,j}. \tag{7}$$

The above equation corresponds to separate $M/2$ paths with delay and $M/2$ paths without delay and construct the Givens rotation matrix from each path with delay to each path without delay.

Also the matrix $\mathbf{X}_0$ is an arbitrary $M \times M$ orthogonal matrix and includes $M(M-1)/2$ free parameters. On the other hand, the matrix $\mathbf{X}_k$ except for $\mathbf{X}_0$ includes $(M/2)^2$ free parameters. Therefore the number of free parameters of [10] is $(K-1)M^2/4 + M(M-1)/2$ and the same as [7]. However the number of adder and multiplication is less than [7].

3. PARAUNITARY FILTERBANKS BASED ON HOUSEHOLDER MATRIX

3.1 Householder Matrix Factorization

An $M \times M$ orthogonal matrix is factorized into $(M-1)$ Householder matrices [1]. A Householder matrix is represented as

$$\mathbf{H}[\mathbf{p}] = \mathbf{I} - 2\mathbf{p}\mathbf{p}^{\dagger} \text{ where } \parallel \mathbf{p} \parallel = 1 \tag{8}$$

where $\mathbf{p} = [p_0, p_1, \cdots, p_{M-1}]^T$ and $(\mathbf{H}[\mathbf{p}])^{-1} = \mathbf{H}[\mathbf{p}]$.

Any orthogonal matrices can be always factorized into cascading Householder matrices. In addition, $\mathbf{H}[\mathbf{p}_0]$ which satisfies following equation is exists [1]:

$$\mathbf{H}[\mathbf{p}_0]\mathbf{X}^{(M)} = \left[\begin{array}{c|c} \mathbf{I}^{(1)} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{X}^{(M-1)} \end{array}\right] \tag{9}$$

where $\mathbf{X}^{(M-1)}$ is $(M-1) \times (M-1)$ orthogonal matrix. By calculating recursively like (9), we can derive the relationship

$$\mathbf{H}[\mathbf{p}_{M-2}] \cdots \mathbf{H}[\mathbf{p}_1]\mathbf{H}[\mathbf{p}_0]\mathbf{X}^{(M)} = \mathbf{I}. \tag{10}$$

Hence, $\mathbf{X}^{(M)}$ can be factorized into

$$\mathbf{X}^{(M)} = \mathbf{H}[\mathbf{p}_0]\mathbf{H}[\mathbf{p}_1] \cdots \mathbf{H}[\mathbf{p}_{M-2}]. \tag{11}$$

Note that the vectors $\mathbf{p}_i$ have the following form:

$$\begin{bmatrix} | & & | \\ \mathbf{p}_0 & \cdots & \mathbf{p}_{M-2} \\ | & & | \end{bmatrix} = \begin{bmatrix} \times & 0 & \cdots & 0 \\ \times & \times & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ \times & \times & \cdots & \times \\ \times & \times & \cdots & \times \end{bmatrix} \tag{12}$$

where each $\times$ denotes an arbitrary value.

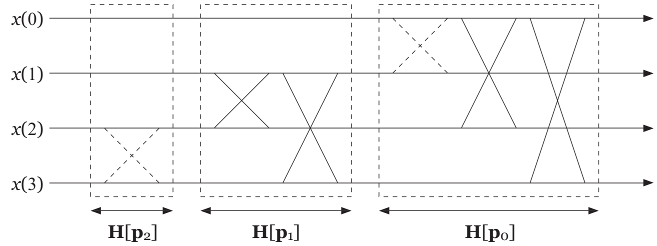

Figure 3: Relationship between Givens rotation and Householder matrices ($M = 4$).

3.2 Relationship between Givens Rotation and Householder Matrices

Firstly, we show the Householder matrix factorization until $\mathbf{H}[\mathbf{p}_{M-3}]$ in (11) to find out a relationship between Givens rotation and Householder matrices.

$$\mathbf{X}^{(M)} = \mathbf{H}[\mathbf{p}_0] \cdots \mathbf{H}[\mathbf{p}_{M-3}] \left[\begin{array}{c|c} \mathbf{I}^{(M-2)} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{X}^{(2)} \end{array}\right] \tag{13}$$

To compare with (11) and (13) and use Givens rotation matrix factorization for $\mathbf{X}^{(2)}$, $\mathbf{H}[\mathbf{p}_{M-2}]$ is rewritten as

$$\mathbf{H}[\mathbf{p}_{M-2}] = \left[\begin{array}{c|cc} \mathbf{I}^{(M-2)} & \multicolumn{2}{c}{\mathbf{0}} \\ \hline \mathbf{0} & \cos\theta_1 & -\sin\theta_1 \\ & \sin\theta_1 & \cos\theta_1 \end{array}\right]. \tag{14}$$

Secondly, we also show the Householder matrix factorization until $\mathbf{H}[\mathbf{p}_{M-4}]$.

$$\mathbf{X}^{(M)} = \mathbf{H}[\mathbf{p}_0] \cdots \mathbf{H}[\mathbf{p}_{M-4}] \left[\begin{array}{c|c} \mathbf{I}^{(M-3)} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{X}^{(3)} \end{array}\right] \tag{15}$$

To compare with (11), (14) and (15) and based on Givens rotation matrix factorization of $\mathbf{X}^{(3)}$, $\mathbf{H}[\mathbf{p}_{M-3}]$ is rewritten as

$$\mathbf{H}[\mathbf{p}_{M-3}] = \prod_{j=M-1}^{M} \mathbf{\Theta}_{M-2,j}. \tag{16}$$

Finally, a relationship between Givens rotation and Householder matrices is represented as

$$\mathbf{H}[\mathbf{p}_i] = \prod_{j=i+2}^{M} \mathbf{\Theta}_{i+1,j}. \tag{17}$$

3.3 Elimination of Redundant Parameters

For simplicity, we consider the case of $M = 4$. A relationship between Givens rotation and Householder matrix factorization is shown in Fig. 3. In this figure, Two redundant Givens rotation angles are drawn as dotted lines. Of course they can be removed, thus simpler Householder matrices can be calculated. The process is presented as follows:

(i) $\mathbf{H}[\mathbf{p}_2]$ is rewritten as $\mathbf{H}[\mathbf{p}_2] = \mathbf{I}$, because the Givens rotation angle can be removed.

(ii) $\mathbf{H}[\mathbf{p}_0]$ does not use $x(1)$. Therefore, $\mathbf{p}_0$ becomes $\mathbf{p}_0 = [p_{0,1}, 0, p_{0,3}, p_{0,4}]^T$.

We can generalize its structure to the M-channel case easily, so (12) is rewritten as

$$\left[\begin{array}{ccc} | & & | \\ \mathbf{p}_0 & \cdots & \mathbf{p}_{M/2-1} \\ | & & | \end{array}\right] = \left[\begin{array}{cccc} \times & 0 & \cdots & 0 \\ 0 & \times & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \times \\ \hline \times & \times & \cdots & \times \\ \vdots & \vdots & \ddots & \vdots \\ \times & \times & \cdots & \times \end{array}\right]. \tag{18}$$

However $\mathbf{p}_0$ in the first block of PUFBs is same as (12). Hence, the number of free parameters of this novel structure is $(K-1)M^2/4 + M(M-1)/2$, same as [10]. We use this structure for the lifting factorization described next section.

4. LIFTING-BASED PARAUNITARY FILTERBANKS

4.1 Lifting Factorization of PUFBs

It is well-known that a Givens rotation matrix is factorized into a lifting structure as [5]

$$\left[\begin{array}{cc} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{array}\right] = \left[\begin{array}{cc} 1 & \alpha \\ 0 & 1 \end{array}\right]\left[\begin{array}{cc} 1 & 0 \\ \beta & 1 \end{array}\right]\left[\begin{array}{cc} 1 & \alpha \\ 0 & 1 \end{array}\right] \tag{19}$$

where $\alpha = (\cos\theta - 1)/\sin\theta$, $\beta = \sin\theta$. Hence PUFBs based on Givens rotation matrix can be factorized into the lifting structures by using (19).

On the other hand, a Householder matrix is also factorized into a lifting structure as [9]

$$\begin{aligned} \mathbf{H}[\mathbf{p}] &= \left[\begin{array}{ccccccc} 1 & & & & & & \\ & \ddots & & & & & \\ & & 1 & & & & \\ \alpha_1 & \cdots & \alpha_{r-1} & 1 & \alpha_{r+1} & \cdots & \alpha_M \\ & & & & 1 & & \\ & & & & & \ddots & \\ & & & & & & 1 \end{array}\right] \\ &\times \left[\begin{array}{ccccc} 1 & & \beta_1 & & \\ & \ddots & \vdots & & \\ & & 1 \;\; \beta_{r-1} & & \\ & & -1 & & \\ & & \beta_{r+1} \;\; 1 & & \\ & & \vdots & \ddots & \\ & & \beta_M & & 1 \end{array}\right] \\ &\times \left[\begin{array}{ccccccc} 1 & & & & & & \\ & \ddots & & & & & \\ & & 1 & & & & \\ \overline{\alpha}_1 & \cdots & \overline{\alpha}_{r-1} & 1 & \overline{\alpha}_{r+1} & \cdots & \overline{\alpha}_M \\ & & & & 1 & & \\ & & & & & \ddots & \\ & & & & & & 1 \end{array}\right]. \end{aligned} \tag{20}$$

Where α_k, β_k and $\overline{\alpha}_k$ are $\alpha_k = p_k/p_r$, $\beta_k = -2p_kp_r$ and $\overline{\alpha}_k = -\alpha_k$, respectively. And $r = i+1$ in (12), (18). Since PUFBs based on Householder matrix can be factorized into the lifting structures using (20).

4.2 Reducing of Rounding Operators

The number of rounding operators should be as small as possible for lossless image coding. For this motivation, we change the position of the Givens rotation angles and merge rounding operators.

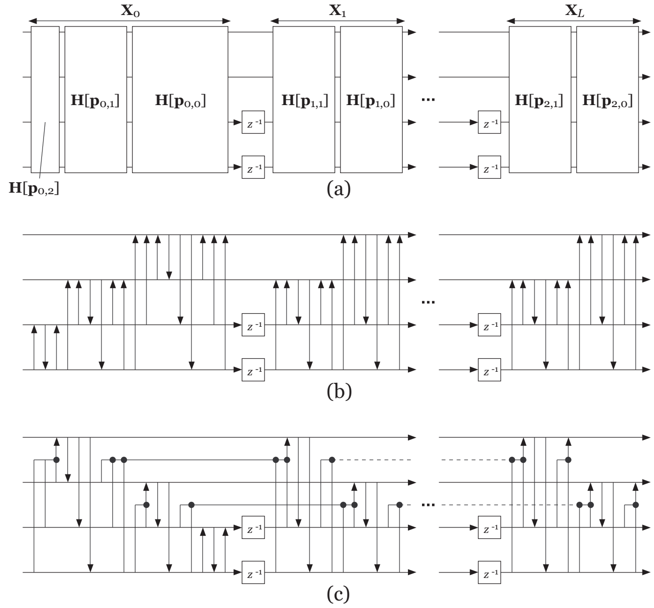

Figure 4: Lattice structures of LBPUFB ($M = 4$): (a)Householder matrix factorization, (b)Lifting factorization, (c)Reducing of rounding operators.

The Householder factorization is also a minimal structure of an orthogonal matrix, since we change the position of the Householder matrices and merge rounding operators. Fig. 4 shows its structure.

The number of reduced rounding operators is shown in Table 1. It is obvious that LBPUFBs based on Householder matrices always have less number of rounding operators than Givens rotation matrix ones. Therefore, we use LBPUFBs based on Householder matrix as our proposed method.

5. RESULTS

5.1 Filterbank Design

In this paper, we focus on image coding applications, thus the cost function is a weighted linear combination of coding gain C_{CG}, stopband attenuation C_{STOP}, DC leakage C_{DC} and a new function C_{abs}. First three functions are very common [1], and C_{abs} is the sum of absolute values of parameters to obtain better compression efficiency.

$$C = -w_1C_{CG} + w_2C_{STOP} + w_3C_{DC} + w_4C_{abs}. \tag{21}$$

We designed 4×8, 4×12, 4×16, 8×16, 8×24 and 8×32 LBPUFBs. Their frequency responses are depicted in Fig. 5.

5.2 Application to Lossless Image Coding

In this subsection, our LBPUFBs are applied to lossless image coding by using rounding operators in each lifting structure. For the fair comparison against WT, we adopted the periodic extention and a very common wavelet-based coder EZW-IP [6]. The coding results are compared by entropy [bpp]=(Total number of bits [bit])/(Total number of pixels [pixel]). Table 2 shows the comparison between LBPUFBs and 5/3-tap WT [8]. In this table, it is obvious that our LBPUFBs denote better results on the entropy than 5/3-tap WT.

5.3 Application to Lossy Image Coding

In this subsection, our LBPUFBs are applied to lossy image coding without using rounding operators. As well as lossless image coding, we adopted the periodic extention and EZW-IP. The coding results are compared by PSNR [dB]=10log$_{10}$(255^2/MSE) where MSE is the mean squared error. Fig. 6 and Table 3 show the part of the

Table 1: The number of reduced rounding operators in LBPUFBs ((·) is the number of rounding operators before reducing).

	4×8	4×12	4×16	8×16	8×24	8×32
LBPUFBs based on Givens rotation matrix	23 (30)	31 (42)	39 (54)	95 (132)	127 (180)	159 (229)
LBPUFBs based on Householder matrix	18 (30)	24 (42)	30 (54)	62 (132)	82 (180)	102 (229)

Table 2: Comparison of lossless image coding (Entropy [bpp]).

Test image (512×512)	5/3-tap WT	4×8 LBPUFB	4×12 LBPUFB	4×16 LBPUFB	8×16 LBPUFB	8×24 LBPUFB	8×32 LBPUFB
Barbara	4.87	4.88	**4.79**	**4.79**	4.88	4.81	4.83
Boat	5.10	5.14	5.10	**5.09**	5.16	5.13	5.15
Elaine	5.11	5.17	5.12	5.11	5.12	**5.06**	5.07
Finger	5.84	5.82	5.74	5.72	5.70	**5.68**	**5.68**
Finger2	5.60	5.63	5.50	5.47	5.48	**5.43**	**5.43**
Grass	6.06	6.11	6.07	6.06	6.07	**6.05**	6.06

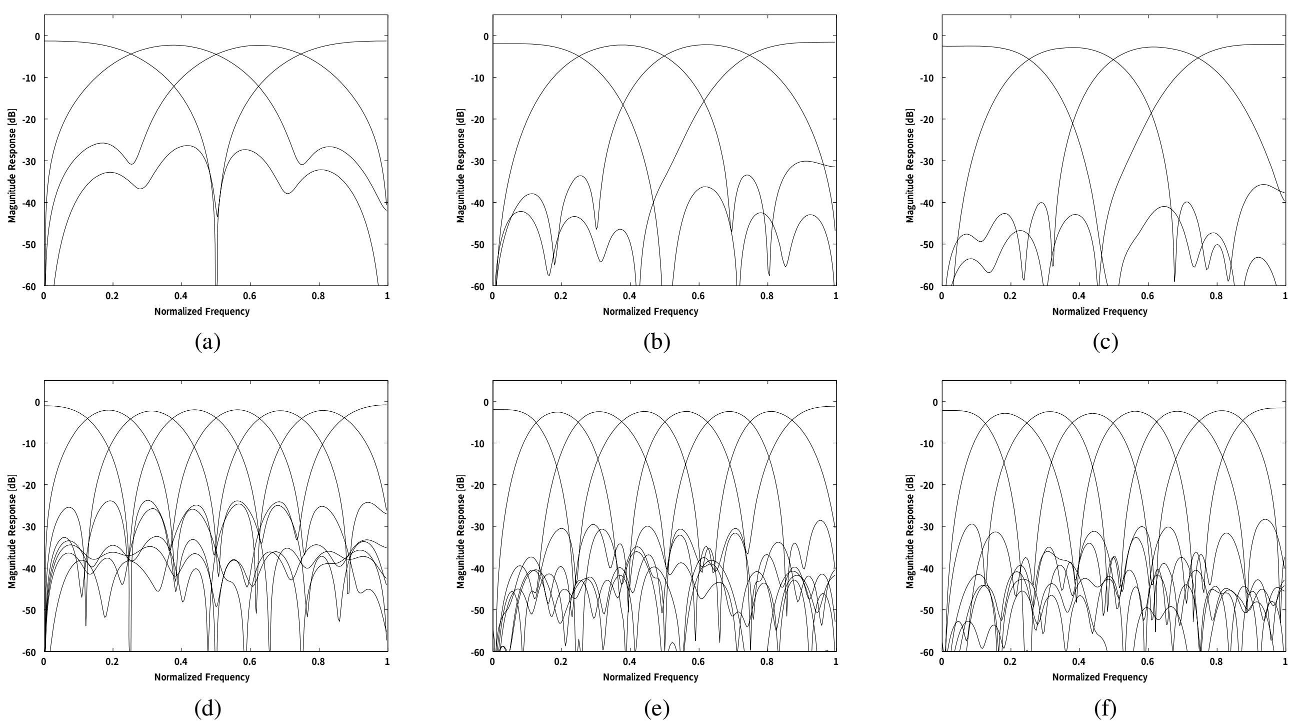

Figure 5: Frequency responses of LBPUFBs: (a)4×8, (b)4×12, (c)4×16, (d)8×16, (e)8×24 and (f)8×32 LBPUFB.

Figure 6: Barbara (Bit rate: 0.25 [bpp]): (left)Original, (middle)9/7-tap WT, (right)8×32 LBPUFB.

Table 3: Comparison of lossy image coding (PSNR [dB]).

Bit rate: 1.0 [bpp]							
Test image (512×512)	9/7-tap WT	4×8 LBPUFB	4×12 LBPUFB	4×16 LBPUFB	8×16 LBPUFB	8×24 LBPUFB	8×32 LBPUFB
Barbara	34.91	35.15	35.71	35.91	35.84	**36.33**	**36.33**
Boat	34.63	34.57	34.79	**34.89**	34.65	34.84	34.82
Elaine	34.89	34.59	34.94	35.01	35.10	**35.52**	35.50
Finger	29.04	29.30	29.74	29.83	30.13	30.36	**30.60**
Finger2	31.26	29.70	30.83	31.51	32.09	32.52	**32.56**
Grass	28.68	28.85	29.04	29.11	29.14	**29.23**	**29.23**
Bit rate: 0.5 [bpp]							
Test image (512×512)	9/7-tap WT	4×8 LBPUFB	4×12 LBPUFB	4×16 LBPUFB	8×16 LBPUFB	8×24 LBPUFB	8×32 LBPUFB
Barbara	30.47	30.66	31.21	31.45	31.56	32.03	**32.07**
Boat	31.44	31.15	31.34	**31.45**	31.27	31.44	31.44
Elaine	33.05	32.71	33.02	33.08	33.04	**33.43**	33.41
Finger	25.96	25.68	26.08	26.22	26.50	26.60	**26.68**
Finger2	27.55	25.68	26.58	26.91	27.15	27.72	**27.90**
Grass	26.11	26.11	26.26	26.34	26.37	**26.47**	**26.47**
Bit rate: 0.25 [bpp]							
Test image (512×512)	9/7-tap WT	4×8 LBPUFB	4×12 LBPUFB	4×16 LBPUFB	8×16 LBPUFB	8×24 LBPUFB	8×32 LBPUFB
Barbara	27.23	27.10	27.62	27.85	27.97	28.40	**28.42**
Boat	**28.47**	28.09	28.32	28.39	28.32	28.44	28.43
Elaine	**31.57**	30.98	31.32	31.39	31.11	31.40	31.37
Finger	23.50	22.87	23.19	23.32	23.65	**23.84**	23.75
Finger2	24.17	22.51	23.32	23.67	23.84	**24.65**	24.57
Grass	24.36	24.17	24.34	24.41	24.48	**24.58**	24.57

enlarged images of *Barbara* and the comparison of PSNRs between LBPUFBs and 9/7-tap WT [8], respectively. In this table and figure, it is obvious that our LBPUFBs denote better results on the PSNR against 9/7-tap WT and the high frequency region of the recnstructed image using the LBPUFB is well-approximated.

6. CONCLUSION

We proposed a novel lattice structure of PUFBs based on the Householder factorization without redundant parameters. This class of FBs, called LBPUFBs, could be factorized into the lifting structures. Our LBPUFBs have less number of rounding operators than the Givens rotation ones and this property is useful for lossless image coding. Furthermore, our LBPUFBs presented superior coding results on the entropy and the PSNR against 9/7- and 5/3-tap WTs to both lossy/lossless image coding, respectively.

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