A NOVEL DESIGN OF CRITICALLY SAMPLED CONTOURLET TRANSFORM AND ITS APPLICATION TO IMAGE CODING

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ABSTRACT

In this paper, a novel design method of critically sampled contourlet transform (CSCT) is proposed. Although, several types of CSCT have been proposed, they have some problems on efficiency and flexibility of their frequency plane partition patterns. In contrast to the way in conventional design methods based on a "top-down" approach, the proposed one is based on a "bottom-up" one. That is, the proposed CSCT decomposes the frequency plane into small directional subbands, and then synthesizes them up to a target frequency plane partition, while the conventional ones decompose into it directly. By this way, the proposed CSCT can provide an efficient and flexible frequency plane partition for image coding.

Index Terms— Critically sampled contourlet, quadrant filter banks, parallelogram filter banks, wavelet transform, image coding

1. INTRODUCTION

Recently, contourlet transform (CT), a multiresolution and multidirectional transform, has been paid much attention in image processing [1]. Unlike the 2-dimensional wavelet transform (2DWT) which arises from tensor products of 1D wavelets, the CT can effectively capture the geometrical features; directional lines, smooth curves, and fine textures in natural images, by cascading some 2D filter banks. Thanks to powerful directional selective capability, the CT can successfully work in several image processing applications [1]. However, the CT-based effective image coding has not established yet, due to the redundancy.

Image coders require critically sampled (CS) CTs rather than oversampled CTs, because the number of samples should not increase after the transform. Up to now, several types of the CSCT have been proposed [2][3]. However, they still have some problems on frequency plane division and filter design for image coding due to the critical sampling constraint, which will be denoted in Section 2. Therefore, the conventional CSCTs are not effective for image coding practically.

In this paper, we introduce a novel design method of the CSCT for image coding. While the conventional design methods take a "top-down" approach, we introduce a "bottom-up" one. This means that the proposed CSCT first decomposes the frequency plane into small fragments of directional subbands, then synthesizes them up to a target frequency plane division, whereas the conventional CSCT decomposes into desired frequency plane partition directly. By this way, flexibility of design is improved and an efficient frequency plane partition for image coding can be realized. Moreover, the proposed CSCT consists of only two kinds of 2D filters: quadrant filter and parallelogram one [4] whose required frequency responses can be easily obtained [5]. Hence, the CSCT can be designed easily and applied for image coding practically.

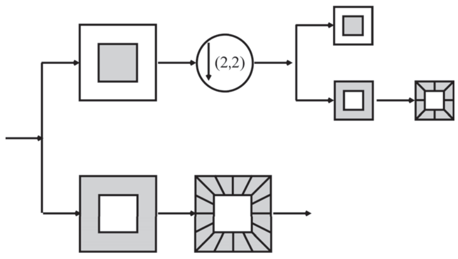
In the following, we organize this paper as follows. In Section 2, we briefly review the conventional CSCTs and denote their problems for image coding. Then, we present the proposed design method in Section 3. In Section 4, we show its effectiveness for image coding. Finally the paper is summarized in Section 5.

Notations: Sampling matrices $\mathbf{R}_0$, $\mathbf{R}_1$ and $\mathbf{D}$ are defined as follows.

$$\mathbf{R}_0 := \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \ \mathbf{R}_1 := \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \ \mathbf{D} := \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}.$$

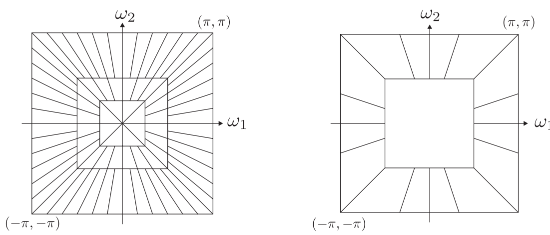
2. CONVENTIONAL CRITICALLY SAMPLED CONTOURLET TRANSFORMS

The CT was initially introduced by Do and Vetterli in [1]. As illustrated in Fig. 1, it consists of Laplacian pyramid (LP) [6] and directional filter bank (DFB) [7]. Since it leads to sparse and directional representation of images, it is efficient for image processing such as denoising and texture analysis [1]. However, it is not suitable for image coding due to its redundancy. The oversampling ratio is equal to $4/3$.



(b)

Fig. 1. Contourlet transform. First, an original image is performed by the LP, then the DFB is applied to highpass subband.



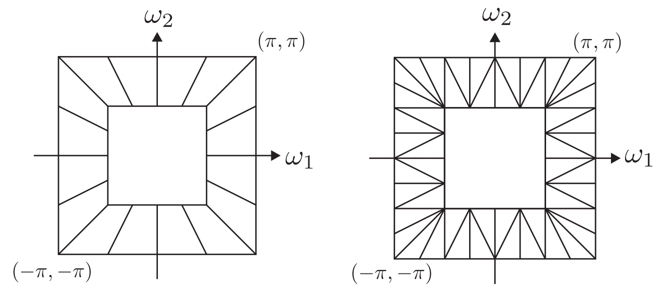
(a) (b)

Fig. 2. Frequency plane partitions in conventional CSCTs: (a) [2], (b) [3]

Recently, several types of critically sampled CTs have been proposed [2][3]. The original CSCT named CRISP-contourlet has been introduced by Do [2]. Its frequency plane partition is given in Fig. 2(a). It can provide multiresolution and directional decomposition with wedge shapes. However, DC energy leaks into some subbands and, thus, energy compaction is degraded, which is a disadvantage for image coding. After that, another CSCT named multiresolution DFB (MDFB) has been presented by Oraintara [3]. Fig. 2(b) illustrates its frequency plane division. Unlike the CRISP-contourlet, this transform keeps DC energy into its lowpass subband. However, in order to realize this frequency partition, seven subband filters have to be designed based on direct design method which cannot offer the exact perfect reconstruction property to filters [3]. As a consequence, it is not efficient for image coding as well.

3. PROPOSED CRITICALLY SAMPLED CONTOURLET TRANSFORM

In this section, we present a novel design method of CSCT which can realize the frequency plane partition of the original CT (Fig. 3) while achieving critical sampling. Generally, in designing a frequency plane division, one has to carefully choose 2D filters which satisfy alias free condition with respect to the sampling matrices [4]. As mentioned in previous section, it is difficult to design the desired frequency plane partition flexibly, due to critical sampling constraints.



(a) (b)

Fig. 3. Proposed frequency plane partition: (a) frequency partitioning obtained by synthesizing fine directional subbands (b) 1-level decomposition of the proposed CSCT.

The conventional CSCTs take a "top-down" approach, i.e. a desired frequency plane partition is directly obtained by cascading 2D filters. In contrast to the conventional CSCTs, we present a "bottom-up" approach; specifically, the proposed CSCT first decomposes the frequency plane into fragments of directional subbands, and then synthesizes them up to a desired partition. This improves the flexibility of design. The proposed CSCT consists of two kinds of 2D filter banks. One is quadrant filter bank and the other is parallelogram one [4]. These filters can obtain the desired frequency responses easily based on the method [5].

Now, we present the proposed tree structure and the partitioning of 2D frequency plane based on the quadrant filter and the parallelogram filter. Our "bottom-up" operation consists of two stages; decomposition and synthesis. First, we decompose 2D frequency plane into partition Fig. 3 (b). After that, some of the directional subbands are synthesized to shape the desired response as Fig. 3 (a).

3.1. Decomposition stage

In order to obtain the pattern illustrated in Fig. 3 (b), we introduce a tree structure given in Fig. 4 and Fig. 5. First, the 1-level decomposition by the 2DWT is applied for an input image to obtain LL, LH, HL and HH subbands. Then, the parallerogram filter and the quadrant one are applied with the binary tree structures given in Fig. 5 for each LH, HL, and HH subbands. Consequently, the frequency plane division illustrated in Fig. 3 (b) can be obtained. Because the 2DWT and each two-channel filter bank in a tree are critically sampled and perfect reconstruction, the entire CSCT can achieve perfect reconstruction with critical sampling. For further level, the same operation of the first level decomposition is applied for LL subband iteratively.

3.2. Synthesis stage

In this stage, we synthesis the fine directional subbands decomposed in the previous stage to obtain the proposed fre-

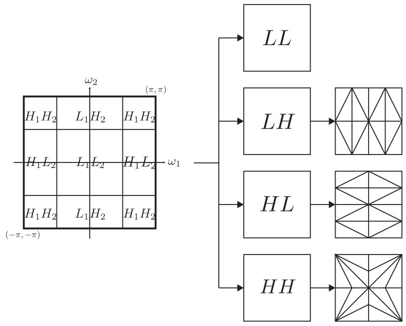


Fig. 4. 1-level decomposition of the proposed CSCT which consists of 1-level 2DWT and LH, HL and HH subband decomposition.

quency division given in Fig. 3 (a). For this purpose, we apply the transmultiplexer [4] shown in Fig. 6 for the fine directional subbands. As shown in Fig. 7, each directional subbands in Fig. 3 (b) can be obtained by synthesizing three finer directional subbands in each corresponding HH, LH and HL subbands, as follows;

$$HH1 + HH2 + LH1 \Rightarrow 1, \quad LH2 + LH3 + LH4 \Rightarrow 2,$$
$$LH5 + LH6 + LH7 \Rightarrow 3, \quad LH8 + HH7 + HH8 \Rightarrow 4,$$
$$HH5 + HH6 + HL8 \Rightarrow 5, \quad HL5 + HL6 + HL7 \Rightarrow 6,$$
$$HL2 + HL3 + HL4 \Rightarrow 7, \quad HH3 + HH4 + HL1 \Rightarrow 8.$$

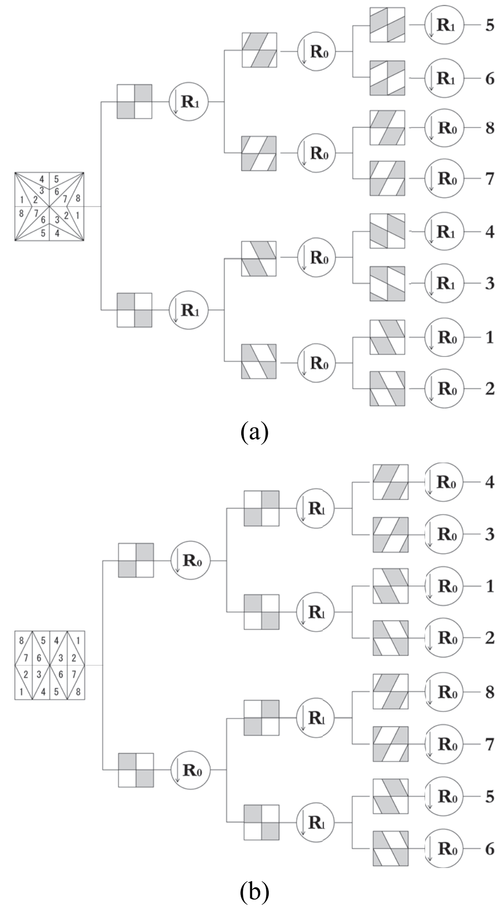


Fig. 5. Binary tree structures: (a) for HH subband (b) for LH and HL subbands.

4. IMAGE CODING

We apply the proposed CSCT to image coding and compare with the 2DWT using the 9/7-tap wavelet filter employed in JPEG2000. As original images, we use "zone plate" and "Barbara" (size: 512×512) which contain large amount of directional information. In this experiment, for the proposed CSCT, 2-level decomposition is applied to the images and then 4-level decomposition of the 2DWT is cascaded to LL subband. As a comparison, 6-level decomposition of the 2DWT is performed to the images. The quadrant filter and the parallelogram one are designed based on the method presented in [5] with the size of 29×29. Their frequency responses are given Fig. 8. In this simulation, the transmultiplexer is set as the delay chain [4], i.e. $H_k(z_1, z_2) = F_{2-k}(z_1, z_2) = z_1^{-k}$ $(k = 0, \ldots, 2)$. EZW-IP [8] is used as a test coder and PSNR was used as an evaluation criterion for reconstructed images. The numerical results are given in Table 1 and the reconstructed images of Barbara at 0.125 bpp are shown in Fig 9. The table indicates that the proposed method is superior to the 2DWT at low bit rate numerically. Moreover, Fig. 9 shows that the directional lines of the reconstructed images obtained by the proposed CSCT are kept much better than those of the 2DWT. Therefore, it can provide rich directional selectivity. As a result, it can be concluded the proposed CSCT provides good coding efficiency in numerical and visual sense.

5. CONCLUSION

In this paper, we proposed a novel design scheme of the CSCT for image coding. Since the conventional "top-down" approach is restricted flexibility of design due to critical sampling constraints, it leads some problems on frequency plane division and filter design. On the other hand, the proposed "bottom-up" approach can contribute a flexible design of frequency plane partition by using easy-feasible 2D filters. Thus the proposed CSCT can realize sparse representation and rich directional selectivity easier than the conventional one. In the image coding experiment, the proposed CSCT outperforms the 2DWT in both numerical and visual sense at low bit rate. Therefore, unlike the conventional CSCTs, the proposed CSCT can be applied to image coding effectively and practically.

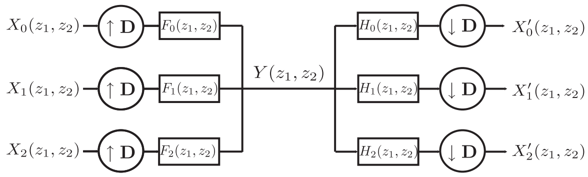


Fig. 6. Transmultiplexer structure.

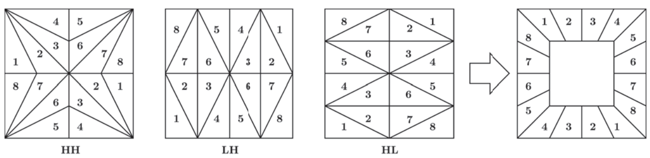


Fig. 7. Proposed bottom-up processing.

6. REFERENCES

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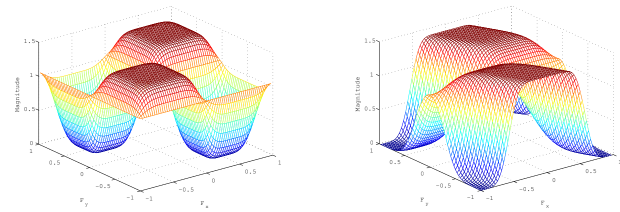


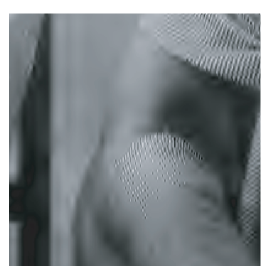
(a) Quadrant filter. (b) Parallelogram filter.

Fig. 8. Design examples of filters (29 × 29) employed in the proposed CSCT.

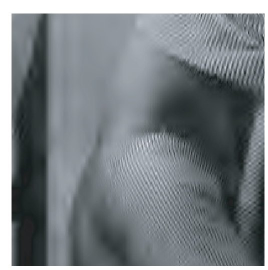
Table 1. Image coding results (PSNR [dB])

Bit rate: 0.25 [bpp]		
	2DWT	Proposed CSCT
Barbara	27.019	27.149
Zone plate	11.824	13.562
Bit rate: 0.125 [bpp]		
	2DWT	Proposed CSCT
Barbara	24.459	24.846
Zone plate	11.105	11.63

(a)



(b)



(c)

Fig. 9. Result image (Bit rate: 0.125 [bpp]) : (a) Original image (Barbara) (b) 2DWT PSNR : 24.459 [dB] (c) The proposed CSCT PSNR : 24.846 [dB]